L3–CALCF

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2006

FORMULAE AND TABLES BOOKLET

This booklet is for use with Calculus 90635, 90636, 90638 and 90639.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

CALCULUS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Some values of $\binom{n}{r}$ are given in the table below.

n \ r	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$$

Complex Numbers

$z = x + \mathrm{i}y$

$\quad = r\ \mathrm{cis}\,\theta$

$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$\quad = r\ \mathrm{cis}\,(-\theta)$

$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre's Theorem:
If n is any integer then
$(r\ \mathrm{cis}\ \theta)^n = r^n\ \mathrm{cis}\ (n\theta)$

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

Circle

$(x-a)^2 + (y-b)^2 = r^2$

has a centre (a,b) and radius r

Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

Focus $(a,0)$ Directrix $x = -a$

Ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

Foci $(c,0)$ $(-c,0)$ where $b^2 = a^2 - c^2$

Eccentricity: $e = \dfrac{c}{a}$

Hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\dfrac{b}{a}x$

Foci $(c,0)$ $(-c,0)$ where $b^2 = c^2 - a^2$

Eccentricity: $e = \dfrac{c}{a}$

CALCULUS

Differentiation

$y = f(x)$	$\dfrac{dy}{dx} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \ \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \ \cot x$
$\cot x$	$-\text{cosec}^2 x$

Integration

$f(x)$	$\int f(x)\ dx$
x^n	$\dfrac{x^{n+1}}{n+1} + c$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Product rule

$(f.g)' = f.g' + g.f'$ or if $y = uv$ then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

Quotient rule

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ or if $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Composite Function or Chain rule

$$\big(f(g)\big)' = f'(g).g'$$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Volume of Revolution

$y = f(x)$ between $x = a$ and $x = b$

rotated about the x-axis

Volume $= \int_a^b \pi y^2\ dx$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x)\,dx \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and n is even.

Parametric Function

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

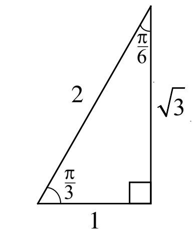
TRIGONOMETRY

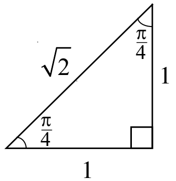
$\text{cosec }\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$





Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Identities

$\cos^2\theta + \sin^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \text{cosec}^2\theta$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

Double Angles

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$\quad = 2\cos^2 A - 1$

$\quad = 1 - 2\sin^2 A$

Products

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Sums

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

MEASUREMENT

Triangle

Area $= \dfrac{1}{2}ab\sin C$

Trapezium

Area $= \dfrac{1}{2}(a+b)h$

Sector

Area $= \dfrac{1}{2}r^2\theta$

Arc length $= r\theta$

Cylinder

Volume $= \pi r^2 h$

Curved surface area $= 2\pi rh$

Cone

Volume $= \dfrac{1}{3}\pi r^2 h$

Curved surface area $= \pi rl$ where $l =$ slant height

Sphere

Volume $= \dfrac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$