UNIVERSITY OF KWAZULU-NATAL
EXAMINATIONS: November 2005
SUBJECT, COURSE AND CODE: MATHEMATICS 001 (PMAOMFY)
DURATION: 3 HOURS TOTAL MARKS: 150

INTERNAL EXAMINERS: MR S PILLAY and MRS J L WARREN
EXTERNAL EXAMINER : MRS A CAMPBELL

Students are requested, in their own interests, to write legibly and in dark blue or black pen.

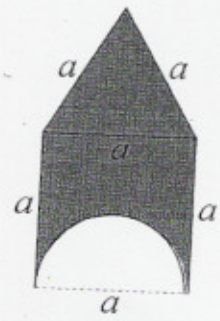
PLEASE NOTE: **This question paper consists of 6 pages. Please see that you have them all.**

- **Show all working in the space provided for each question.**
- **Students on the Pietermaritzburg campus answer all questions except number .**
- **Students on the Westville campus answer all questions except number.**

Question 1:

In the diagram alongside, the curved line is a semicircle. Find:

a) the area of the shaded region (5.5)

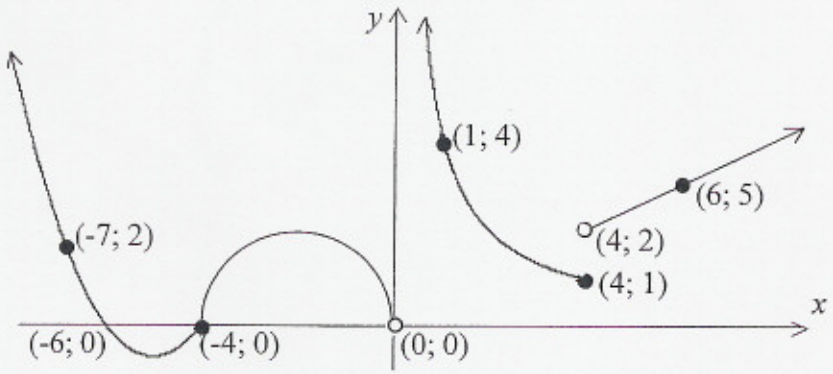


b) the perimeter of the shaded region. (2)

[7.5]

Question 2:

The piecemeal graph shown below, consists of a parabola, a semicircle, a hyperbola and a straight line.



a) Is the graph a function? Explain. (1)

b) Write the domain in interval notation. (2.5)

c) Find the definition of the graph. (8)

d) Write the range in interval notation. (1)

[12.5]

Question 3:

The graph of the quadratic function, $f(x) = 2(x-1)^2 - 1$, is given below.

a) On the same axes, draw the inverse of f. (3)

b) Give a greatest restriction of f, called h, so that the inverse of h will be a function. (1)

c) Find $(h \circ g)(x)$ if $g(x) = \frac{x}{2} - 1$. (3.5)

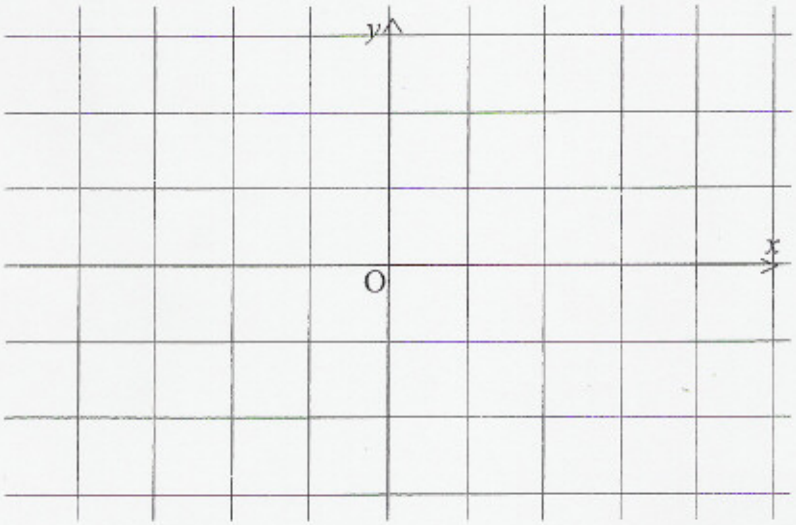
d) Find the equation of the inverse of h. (4)

[11.5]

Question 4:

$$f(x) = -2|x+1|+4$$

a) On the axes provided below, draw the graph of $g(x) = |f(x)|$. (5.5)

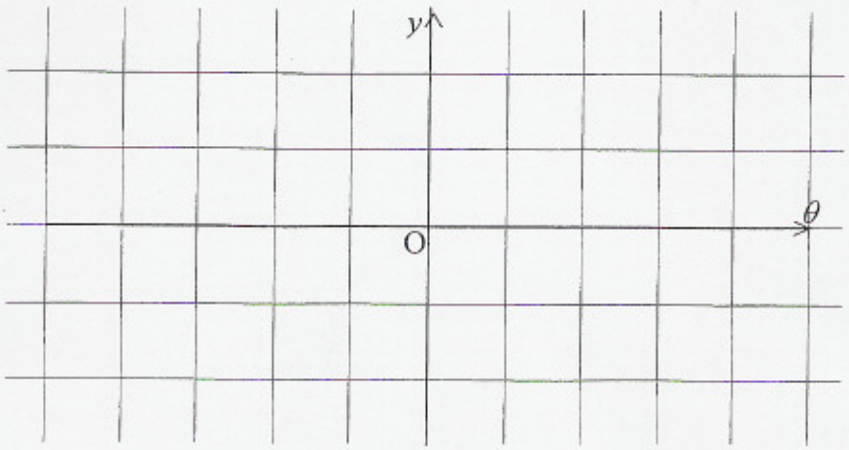


b) Find x if $g(x) = 4$. (2.5)

[8]

Question 5:

a) On the axes provided below, draw graphs of $y = 2\sin\left(\theta - \frac{\pi}{2}\right)$ and $y = |\cos\theta|$ for $\theta \in [-2\pi; 2\pi]$. Show all stages and do not use a calculator. (5)



Question 5 Continued:

b) Use the graph drawn in a) to solve: $|\cos\theta| > 2\sin\left(\theta - \frac{\pi}{2}\right)$, $\theta \in [-2\pi; 2\pi]$.

(3)
[8]

Question 6:

Prove the identity: $2\sin^2\theta = \frac{\sec 2\theta - 1}{\sec 2\theta}$. **[4]**

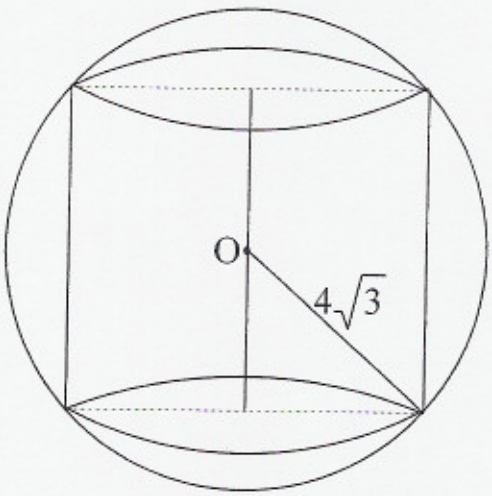
Question 7:

Find the general solution of the equation: $(\sin\theta + \cos\theta)^2 = \frac{2-\sqrt{3}}{2}$. **[6.5]**

Question 8:

Find the height of the cylinder in the diagram alongside. You are told the following:

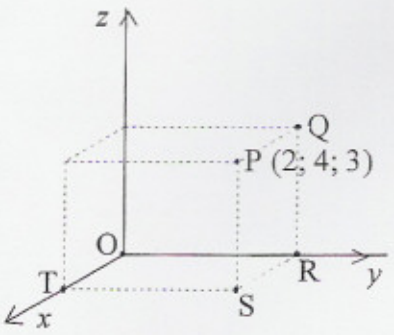
- O is the centre of the sphere.
- The sphere has radius $4\sqrt{3}$ cm.
- The cylinder fits exactly into the sphere.
- The cylinder must have **maximum** volume.



[10]

Question 9:

The box in the diagram given, has one of its corners at the origin, O, and three of its edges lie along the three axes. P is the point (2; 4; 3).



a) The box is rotated 180° about the z-axis.

(i) What are the new coordinates of P? (1.5)

(ii) Find the distance between the original and new positions of P. (2)

(iii) Give the equation of the plane containing the new positions of O, R, S and T. (1)

b) (i) Draw a diagram showing the position of the point $\left(2; \frac{5\pi}{6}\right)$ given in polar coordinates. Label your diagram carefully. (1.5)

(ii) Give an example of a point whose Cartesian coordinates, $(x; y)$, and polar coordinates, $(r; \theta)$, are such that $r = x$ and $\theta = y$. (1)

[7]