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EXTERNAL EXAMINER: Mrs A. Campbell

PLEASE NOTE:      1. The paper consists of 8 pages. Check that you have them all.
     2. Show all working and attempt all questions.
     3. Students are requested, in their own interest, to write legibly.

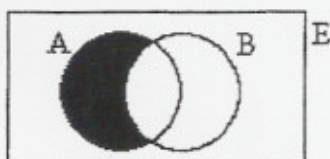SECTION A: SHORT QUESTIONS      20 MARKS

STUDENT NUMBER............................................

**Question**          **Answer**

1. Use the given Venn diagram to write the shaded area in terms of A and B.

1.

2. Let $R$ be the set of real numbers and $Z$ the set of integers. Then $R \cap Z =$

2.

3. Simplify: $|\pi - 4| + 1$. (Give your answer in terms of $\pi$.)

3.

4. Convert $3\,007\ cm^3$ to $m^3$.

4.

5. " There are six times as many students as there are professors at this university." Write an equation for this statement using variables S and P, with S for the number of students and P for the number of professors.

5.

6. Solve the following inequality for $x$: $\dfrac{2x-1}{|x+2|} < 0$.

6.

7. The sketch shows the graph of a function $f$. Sketch the graph of $f^{-1}$.

7.

8. The $y$-intercept for the graph $y = f(x)$ is 4. What is the $y$-intercept for the graph $y = -f(x)+1$?

8.

9. If $0{,}125 = 2^x$, then $x = ...$

9.

10. Simplify: $\dfrac{4^x + 3.2^{2x-1}}{2^{-x}.2^{3x+2}}$

10.

11. Express $\csc\left(\dfrac{5\pi}{3}\right)$ in terms of trig functions of 2 different acute angles.

11.

SECTION B:                                         130 MARKS

## QUESTION 1

(a)   A system of three linear equations in $a$, $b$ and $c$ is solved by Gauss elimination method and we arrive at the step:

| $a$ | $b$ | $c$ | 1 |
|---|---|---|---|
| 1 | 2 | -3 | 6 |
| 0 | 1 | -2 | 2 |
| 0 | 0 | 0 | $k$ |

   (i)     For what values of $k$ is there a unique solution?                          (1)
   (ii)    For what values of $k$ is there no solution?                                (1)
   (iii)   For what values of $k$ are there infinitely many solutions? Find all solutions for such value(s) of $k$.                                                          (5)

(b)   A dietician wants to combine three foods so that the resulting mixture contains 900 units of vitamins, 750 units of minerals and 350 units of fat. The units of vitamins, minerals and fat contained in each gram of the three foods are shown in the table. How many grams of each food should be combined to obtain the required mixture?

|  | Vitamins | Minerals | Fat |
|---|---|---|---|
| 1 gram of food A | 35 units | 15 units | 10 units |
| 1 gram of food B | 10 units | 20 units | 10 units |
| 1 gram of food C | 20 units | 15 units | 5 units |

                                                                                  (11)
                                                                                  [18]

## QUESTION 2

(a)   A right cylinder prism has a volume V.

   (i)     If you multiply the area of the base by $p$, what must happen to the height of the cylinder in order to keep the volume constant?                              (1)
   (ii)    What kind of proportion exists between the area of the base and the height of the cylinder?                                                                    (1)
   (iii)   If you multiply the radius of the base by $p$, what must happen to the height of the cylinder to keep the volume the same?                                     (1)
   (iv)    If you multiply both the radius and the height of the cylinder by $p$, what would happen to the volume of a right cylinder prism?                              (1)

(b)   Three semicircles PQS, QRT and PRQ are given as shown in the diagram below. Show that the sum of the area of the shaded parts is equal to the area of △PQR.
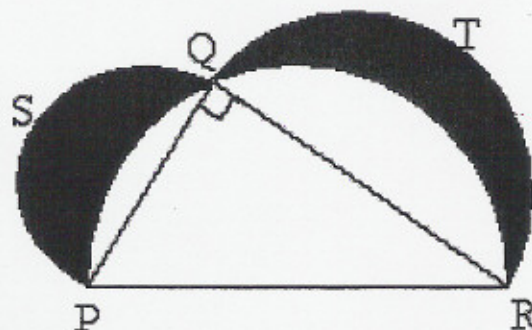


(7)
[11]

## QUESTION 3

(a)   (i)   Convert 175° to radian measure. (Give answer simplified in terms of $\pi$)   (1)
      (ii)   Convert 5 radians to degree measure. (Answer correct to 4 significant figures)   (1)

(b)   In the following figure, O is the centre of the circle and OPQ is a circular sector with radius $r$ cm and central angle $2\theta$ is in radian measure. The perimeter of the sector is 12 cm.
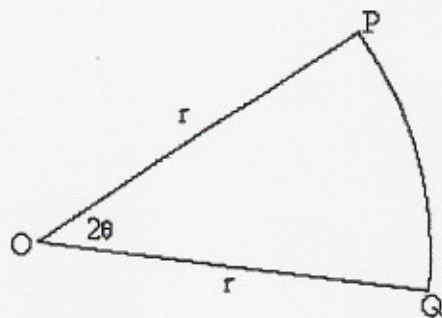


(i)   Show that arclength $PQ = 2r\theta$   (1)
(ii)   Express $r$ in terms of $\theta$.   (3)
(iii)   Express the area A of the sector in terms of $\theta$.   (2)
[8]

## QUESTION 4

In the sketch, the inverses S, T and R are represented by a hyperbola, parabola and a straight line respectively, with points A (-3; 2½),   B (-3; -2½) and  C (2; -2) as shown in the sketch.
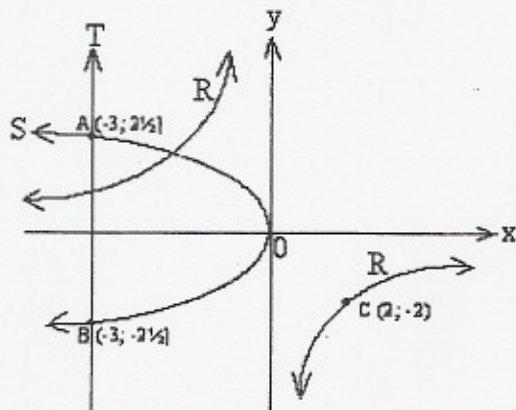


Use the data and write down the following:

(a)   (i)   R in the form $\{ (x; y): y = ... \}$                                         (1)

        (ii)   S in the form $\{ (x; y): y = ... \}$                                        (4½)

(b)   $r$,  $s$   and $t$, where $r$,  $s$   and $t$ are inverses of R, S and T
        respectively, in the form $\{ (x; y): y = ... \}$                                  (3)

                                                                                        [8½]

## QUESTION 5

(a)   Let $f(x) = -\sqrt{9 - x^2}$ and $g(x) = \dfrac{4}{2 - x}$. Find the following:

(i)     $(f \circ g)(-2)$                                                                (2)

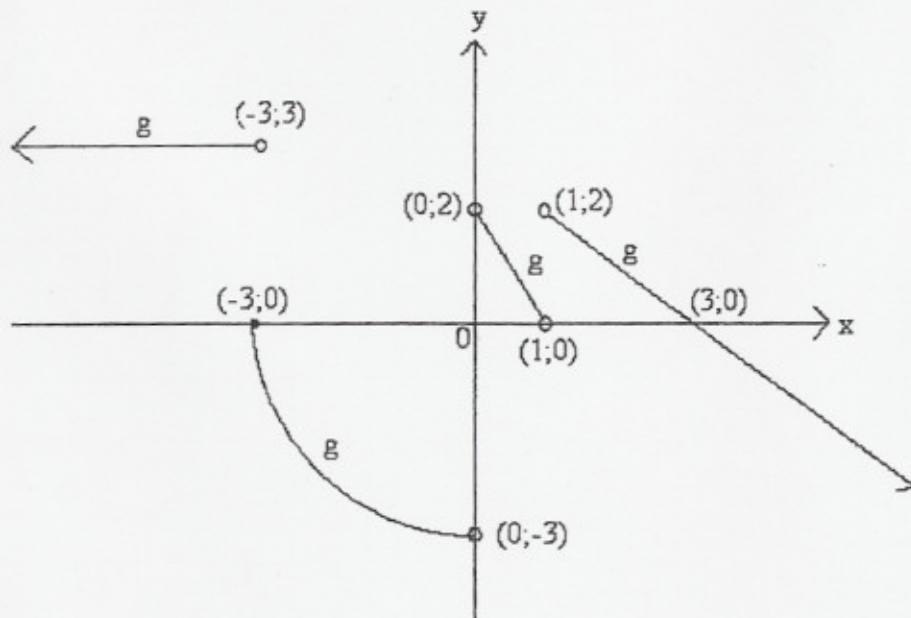(ii)    $(g \circ f)(4)$                                                                 (1½)

(b)     The graph of $g(x)$ is drawn below.



(i)     Write down the definition of $g$.        (6½)

(ii)     From the graph, find:    1.    $g^{-1}(2)$
                                       2.    $g(3)$
                                       3.    $g^{-1}(3)$        (3)

                                                                               **[13]**
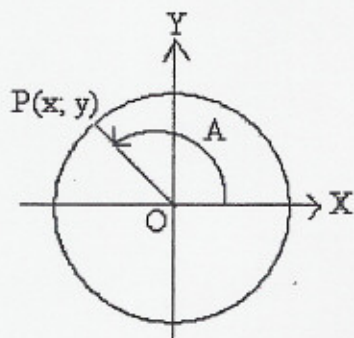
## QUESTION 6

(a)     Draw the graph of the function given by $f(x) = \dfrac{|x|}{x}$        (3)

(b)     Given that $x = -1$, evaluate $|3x - 1| - |-5x|$        (3)

(c)     Solve for $x$:     $5 - |x - 1| > 1$        (4)

                                                                                 **[10]**

## QUESTION 7

(a)    P$(x; y)$ is a point on the circumference of a circle with radius $r$, centre $(0; 0)$ and $X\hat{O}P = A$.



Determine, without using a calculator, the value of A in radians if:

$$x = -\frac{\sqrt{3}}{\sqrt{2}} \quad \text{and} \quad y = \frac{1}{\sqrt{2}}.$$

(3)

(b)    Use reduction formulae to simplify to a single trigonometric ratio:

$$\frac{\tan(2\pi - \theta).\sin\left(\frac{\pi}{2} + \theta\right).\sec\left(\frac{3\pi}{2} + \theta\right)}{\csc\theta.\cot\left(\frac{\pi}{2} - \theta\right).\sin(\pi + \theta)}$$

(6)

(c)    (i)    On the same system of axes draw the graphs of $f$ and $g$, where
             $f(\theta) = \cos\theta$ and $g(\theta) = -2\cos 2\theta$ for $\theta \in [-\pi; \pi]$ .

(4)

       (ii)   **State** how you would obtain the graph of $h$ from the graph of $f(\theta) = \cos\theta$ if

             $h(\theta) = 3\cos\left(\frac{\theta}{2} + \frac{\pi}{2}\right) - 1$. Do not draw the graph of $h(\theta)$.

(4)

(d)    Consider $f(\theta) = \cos\theta.\cos 2\theta + \frac{(\sin 2\theta)^2}{2\cos\theta}$.

       (i)    For what value of $\theta$, $\theta \in R$, is $f(\theta)$ undefined?

(3½)

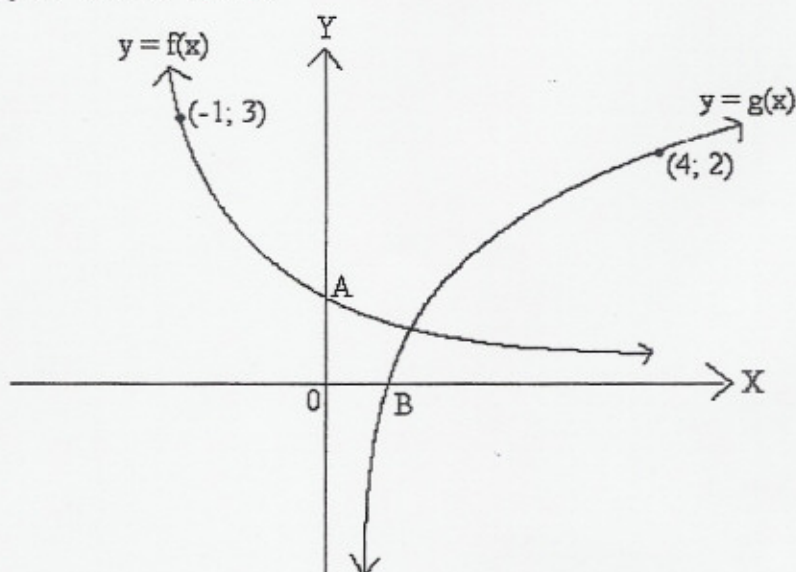       (ii)   Show that $f(\theta) = \cos\theta$.

(5)

[25½]

## QUESTION 8

The figure shows the graphs of $f$ and $g$, where

$$y = f(x) = a^x, \quad a > 0, \quad a \neq 1$$

and

$$y = g(x) = \log_b x, \quad b > 0, \quad b \neq 1.$$



(a)    Determine the values of $a$ and $b$. (2)

(b)    Give the coordinates of A and B. (2)

(c)    If the graph of the function $h$ is a reflection of the graph of $g$ in the X-axis, give the equation of $h$ in the form: $\quad y = h(x) = ....$ (1)

(d)    Solve for $x$: $\quad 3^x = \dfrac{31}{7^x}$ $\quad$ (Give your answer correct to four significant figures.) (5½)

[10½]

## QUESTION 9

(a)    (i)    Let $g(x) = \begin{cases} \alpha x^2 & if \quad x > 2 \\ x+1 & if \quad x < 2 \end{cases}$

What value must $\alpha$ be so that $\lim\limits_{x \to 2} g(x)$ exists? (3)

(ii)    Evaluate the following limit, if it exists:

$$\lim_{t \to -3} \frac{t^2 - 9}{2t^2 + 7t + 3}$$ (2)

(b)    Use the rules of differentiation to determine $\dfrac{dy}{dx}$ if:

    (i)    $y = (x^2 + 1)(x^3 - 1)$    (2)

    (ii)    $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$    (3)

(c)    Find two positive numbers whose sum is 20 and whose product is as large as possible.    (4½)

(d)    Consider the function: $g(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

    (i)    Find the intervals on which $g$ is increasing and decreasing.    (8)

    (ii)    Find the local minimum and maximum values of $g$.    (3)

    [25½]