UNIVERSITY OF KWAZULU-NATAL
NOVEMBER 2008 EXAMINATIONS
COURSE AND CODE: FOUNDATION MATHEMATICS (MATH 199)
07 NOVEMBER 2008

DURATION: 3 HOURS **MARKS: 150**

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EXTERNAL EXAMINER: Dr O. Narain.

PLEASE NOTE:

1. This paper consists of 14 pages, including this cover page. Check that you have them all.
2. Answer all questions and show working details.
3. Write your answers in the space provided. If the provided space is not enough, you can use the reverse side of the previous page, ie. opposite where you are working.
4. Write legibly in dark blue or black pen.
5. **Write your student number in the space provided below.**

STUDENT NUMBER:.....

STUDENT SIGNATURE:.....

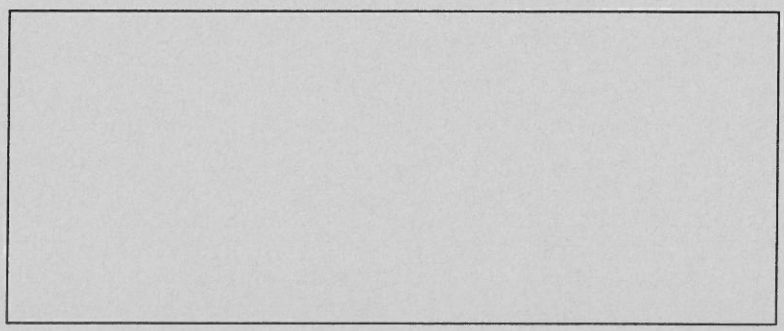
SEAT NUMBER:.....

Question Number	1	2	3	4	5	6	7	8	Total
Total	24	13	18	20	6	22	15	32	150
Internal Marks									
External Marks									

Question 1 (24)

a) i) Illustrate each of the following sets using a Venn diagram.
E is the set of integers between 0 and 14.
T is the set of composite numbers from 1 to 5.
P is the set of prime numbers from 1 to 9.
K is the set of even numbers between 1 and 13.

(4)



ii) Hence determine $n(P' \cap K)$. (2)

b) i) Prove that 0.454545.... is a rational number. (4)

ii) Prove that the sum of two consecutive even numbers is even. (4)

c) If it takes 3 men 7 days to complete a job, determine how long it will take 12 men to complete the same job.
(Assume that the men work at the same rate.)

(4)

d) Solve the following system of linear equations using the Gauss Reduction method. (6)

x	y	z	1
1	2	3	6
0	1	-2	2
2	-1	1	2

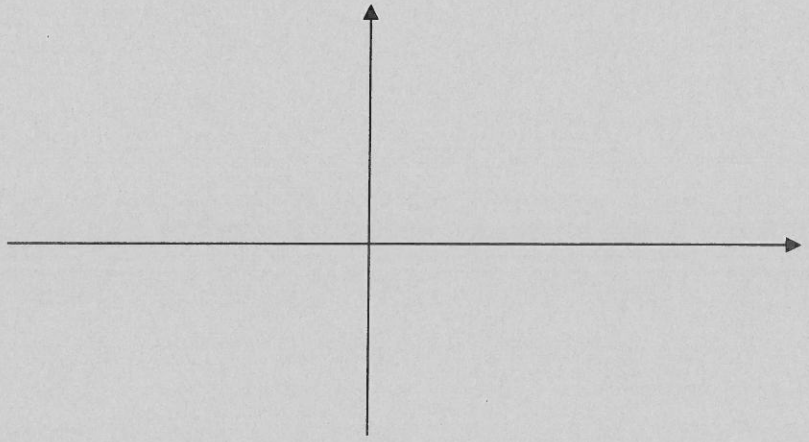
Question 2 (13)

Consider the function f defined by $f(x)=\begin{cases}-4, & -2\le x<0\\ \sqrt{9-x^2}, & 0<x<3\end{cases}$.

a) Evaluate: i) $f(-1)$ (1)

ii) $f(3)$ (1)

b) Sketch the graph of f on the set of axes provided below. (3)



c) Write down the domain of f using interval notation. (3)

d) Will the inverse of f be a function? Explain. (2)

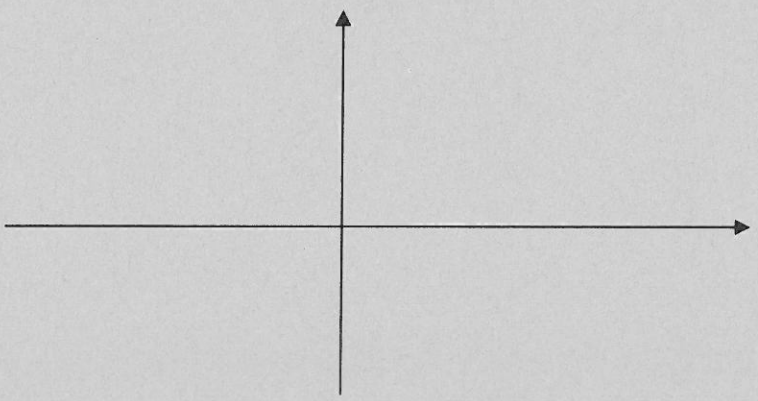
e) Show that $\lim_{x\to 0} f(x)$ does not exist. (3)

Question 3 (18)

Consider $f(x) = 2|x+1| - 3$

$g(x) = (0.25)^x$

a) On the system of axes below, sketch graphs of f and g. (5)



b) Does f have a maximum or minimum value? Give a reason. (2)

c) Give the range of f using interval notation. (1)

d) Shade the region defined

$\{(x, y) \in R^2 : y \geq g(x)\} \cap \{(x, y) \in R^2 : y \leq f(x)\}$. (2)

e) Solve for x algebraically: $2|x+1| - 3 < 5$. (3)

f) Determine x if $g(x) = f(2.5)$. (3)

g) State the inverse of g. (2)

Question 4 (20)

a) Simplify $\dfrac{4^{x+1}+4^{x-1}}{2^{2x}}$ (3)

b) Evaluate $3\ln e^2 + \ln\sqrt{e}$. (3)

c) Determine k if $9^{k+1}\times\left(\dfrac{1}{27}\right)^{2k} = 81$. (4)

d) Solve for x if $\log_x 3 + \log_3 x = 2$. (5)

e) Determine m if $10^{2m} - 101 \times 10^{m} = -100$. (5)

Question 5 (6)

Consider $f(x) = x^2 - 1$ and $g(x) = \log x$.

a) Determine x if $f(x) \geq 0$. (4)

b) Hence determine the domain of $y = (g \circ f)(x)$. (2)

Question 6 (22)

a) Give an angle in the interval $[-720°, -360°)$ which is co-terminal to $40°$. (2)

b) Prove using the definition of trigonometric functions that
$$1 + \tan^2\theta = \sec^2\theta$$ (3)

c) Simplify to a single trigonometric function of θ:
$$\frac{\sin(A - 360°)\tan(90° + A)\cos(360° - A)}{\sin(270° - A)}$$ (4)

d) Evaluate without a calculator:

i) $\dfrac{\sin\left(-\dfrac{\pi}{2}\right)\tan\left(\dfrac{5\pi}{4}\right)\cos\left(-\dfrac{5\pi}{6}\right)}{\cot\left(\dfrac{7\pi}{3}\right)}$ (4)

ii) $(\sin 22.5 + \cos 22.5)^2$ (4)

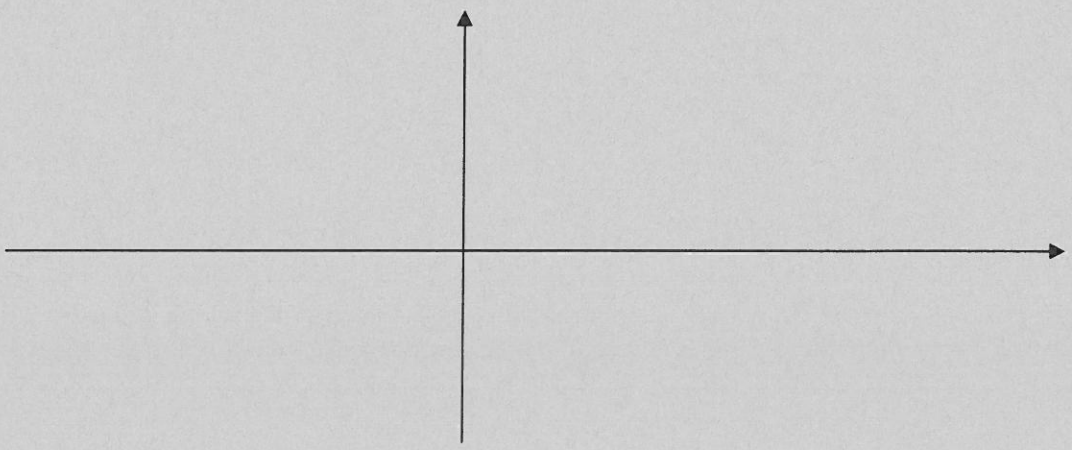
e) Prove $\dfrac{\sin 2x - \cos x}{\sin x - \cos 2x} = \dfrac{\cos x}{\sin x + 1}$. (5)

Question 7 (15)

a) Consider $f(x) = 2\sin 4x, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and

$g(x) = -\tan 2x, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

i) Sketch graphs of f and g on the system of axes below. (6)



ii) Give the range of $y = f(x - \pi)$. (2)

iii) Explain how the graph of g must be shifted in order to obtain the graph of $y = -\tan(2x - \pi)$.

(2)

b) Determine a general solution for θ if $2\sin\theta\cos\theta = -\frac{\sqrt{3}}{2}$.

(Assume θ is measured in degrees.) (5)

Question 8 (32)

a) Consider $f(x) = x^2 - x$.

i) Determine $f'(x)$ using first principles. (5)

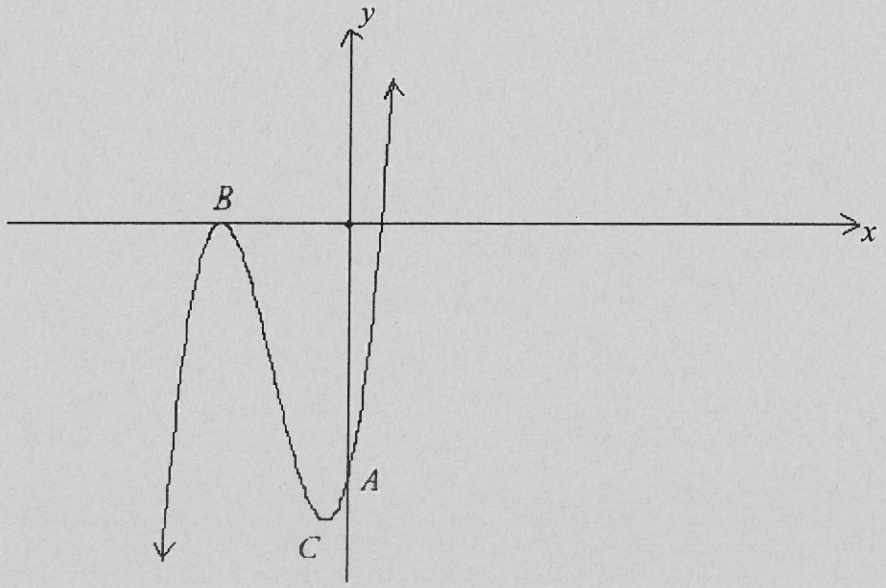
ii) Hence determine the equation of a tangent to f at $x = 2$. (4)

b) Determine :

i) $D_x\left[\dfrac{2x-\sqrt{x}}{3x^2}\right]$ (4)

ii) $\dfrac{dy}{dx}$ if $y=\left(3x-\dfrac{k}{x}\right)^2$, k is a real constant (4)

c) The graph of $y = 2x^3 + 7x^2 + 4x - 4$ is sketched below.

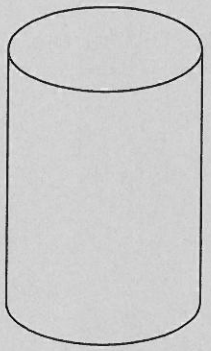


i) Determine the coordinates of A, the y-intercept of the graph. (1)

ii) Determine the coordinates of the stationary points B and C. (4)

iii) State the x-value(s) for which the graph is increasing. (2)

d) A manufacturer wants to make a closed cylindrical can, like the one shown alongside, which must hold 1000 cm^3 of liquid. Assume the can has radius r cm and height h cm.



i) Write down an expression in terms of r and h for the volume of the can.

(1)

ii) Write down an expression in terms of r and h for the total surface area of the can.

(3)

iii) Determine r so that the total surface area of the can is minimised.

(4)