UNIVERSITY OF KWAZULU-NATAL
JUNE 2007 TEST
COURSE AND CODE: FOUNDATION MATHEMATICS (MATH 199)

DURATION: 3 HOURS **MARKS: 150**

INTERNAL EXAMINER : Mr Y.P. Latchmanan
EXTERNAL EXAMINER: Ms. H. Tarr

PLEASE NOTE:

1. This paper consists of 18 pages, including this cover page. Check that you have them all.
2. Show all working and attempt all questions.
3. Write your answers in the space provided. If the provided space is not enough, you can use the reverse side of the previous page, ie. opposite where you are working.
4. Write legibly in dark blue or black pen.
5. **Write your student number in the space provided below.**

STUDENT NUMBER:.....

STUDENT SIGNATURE:.....

SEAT NUMBER:.....

DATE:.....

Question Number	1	2	3	4	5	6	7	8	9	Total
Internal Marks										
External Marks										

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QUESTION ONE [17 marks]

(a) True or False? All natural numbers are rational numbers.
Give a reason for your answer. (1)

(b) Let M be a set of real numbers between -6 and 10. Illustrate on separate number lines, the set of all elements in M that are:

(i) negative integers. (2)

(ii) prime numbers. (2)

(iii) real numbers greater than -1 but less than or equal to 5. (2)

(c) Define a rational number and hence, show that $0.58\dot{3}$ is a rational number. (5)

(d) Prove that the product of two odd numbers is an odd number. (5)

QUESTION TWO [18 marks]

(a) Let B be any non-empty set, and A be a subset of B.

State whether each of the following statements are true or false. If false give the correct answer:

(i) $A \cup A' = \{\ \}$ (2)

(ii) $A \cap \{\ \} = B$ (2)

(b) Consider the following sets of different make of motor vehicles which were recently auctioned by the Department of Transport, where E is the universal set.

$E = \{$ Corsa, Polo, Corolla, Mazda, Astra, Golf, Hilux, Nissan, Isuzu, Bantam $\}$

$G = \{$ Astra, Polo, Nissan, Bantam $\}$

$P = \{$ Bantam, Corolla, Polo, Golf, Corsa $\}$

$Q = \{$ Golf, Polo, Corolla $\}$

(i) Draw a Venn diagram, showing all the sets given above, and put the elements in their correct places. (5)

(ii) Determine:

$$(Q \cap P)' \cap G$$ (3)

(iii) Determine:

$$n(G \cup G')$$ (3)

(iv) On your Venn diagram in Question b(i) above, shade $Q' \cap (G \cup P)$. (3)

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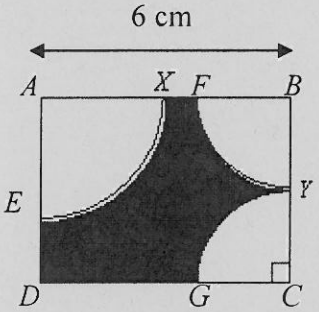
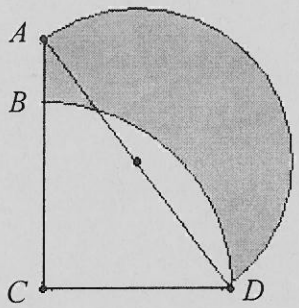
QUESTION THREE [28 marks]

(a) In the diagram below, all curved lines are arcs of circles. $ABCD$ is a rectangle with $AB = 6$ cm, $AX = XB$ and $BY = YC = 2$ cm. Determine the perimeter of the shaded region. (6)



(b)

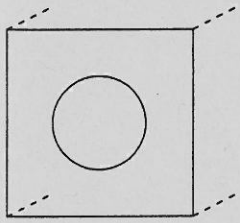


In the diagram alongside,

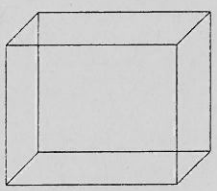
- ACD is a right-angled triangle with $A\hat{C}D = 90°$
- $AC = 28$ cm and $BC = 21$ cm.
- A semicircle is drawn using AD as diameter.
- Sector BCD has CD as radius.

Determine the area of the shaded region.
Leave your answer in terms of π. (7)

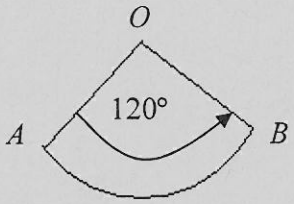
(c) A solid wooden cube has side of length 5 cm. A circular hole of diameter 1 cm is drilled through **one of the faces of the wooden cube** as shown below. The hole is 2 cm deep. Determine the total surface area of the wooden cube. (8)



(d)



Prism 1



Cross Section of Prism 2

Prism 1, which is a rectangular prism, has a length of $2x$ mm, a breadth of x mm and a height of h_1 mm.

Prism 2, whose cross section is a sector with radius x mm, has angle AOB $= 120^0$.

Prism 1 is completely full of water. All the water from prism 1 is then poured into prism 2. The water level in prism 2 is h_2 mm.

Determine $h_2 : h_1$. (7)

QUESTION FOUR [16 marks]

(a) State the type of proportion between x and y, in each of the following (give a reason in each case) :

(i)

x	2	4	- 8
y	3	6	- 12

(2)

(ii)

x	1	3	5
y	2	4	6

(2)

(iii)

x	- 1	- 4	- 6
y	12	3	2

(2)

(b) Jane is 25 years younger than her mother. In 12 years time she will be half her mothers' age. Calculate Jane's age currently, by making use of suitable equations. (5)

(c) Two cyclists are 80 km apart and travel towards each other. They start at 7 :00 am and cycle at a constant speed of 15 km/h and 10 km/h, respectively. By using suitable equations determine at what time the cyclists will meet.

(5)

QUESTION FIVE [6 marks]

(a) Determine the solution to the following system of equations, using Gauss Reduction.

$$\begin{aligned} x + 2y - z &= 1 \\ 5x - y + 2z &= 5 \\ 2x + 4y - 2z &= 2 \end{aligned}$$

(6)

QUESTION SIX [12 marks]

(a) Solve for x and illustrate your solution graphically:

$$-x+1 \leq \frac{3+2x}{2} < 5 \qquad (5)$$

(b) Solve for x and write your solution in interval notation:

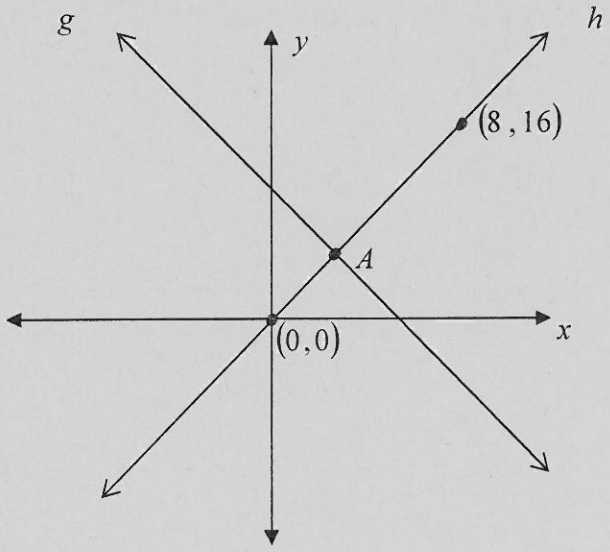
$$\frac{1}{x-3} \leq \frac{9}{4x+3} \qquad (7)$$

QUESTION SEVEN [14 marks]

(a) Find the domain, in set-builder notation, of the following function:

$$f(x) = \frac{1}{\sqrt{2 - x}}$$ (3)

(c) The graphs of $g(x) = -x + 6$ and $h(x)$ are illustrated on the diagram below:



(i) Determine the equation of h. (3)

(ii) Determine the co-ordinates of point A, ie. the point of intersection of g and h. (2)

(iii) Shade on the diagram the region represented by $\{(x,y) \in R^2 \mid y \geq g\} \cap \{(x,y) \in R^2 \mid y \geq h\}$. (2)

(iv) Show that $(g^{-1} \circ g)(x) = (g \circ g^{-1})(x) = x$. (4)

QUESTION EIGHT [12 marks]

Consider the function f below:

$$f(x) = 3x^2 + 2x - 1$$

(a) Write $f(x)$ in the form $y = a(x-p)^2 + q$. (3)

(b) Find the equation of the inverse of f. (3)

(c) Write down the greatest restriction of f, called g, so that the inverse of g is a function. (2)

(d) Sketch the region defined by:

$$A = \{(x,y) \in R^2 \mid \ x \geq 0 \ \} \cap \{(x,y) \in R^2 \mid \ y \geq f(x)\}$$ (4)

QUESTION NINE [27 marks]

(a) State if the following are **TRUE** or **FALSE.**

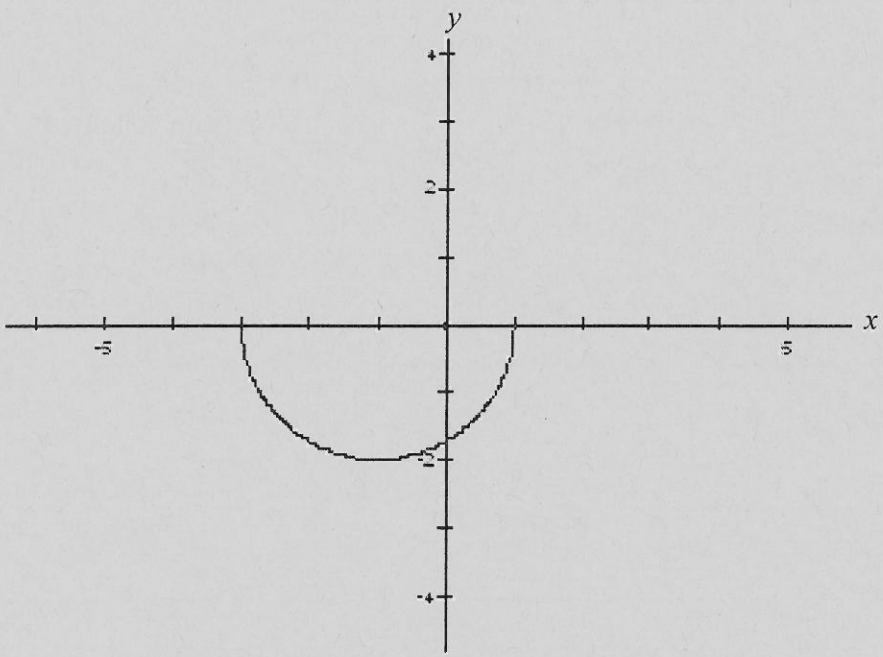
(i) A circle is a function. (1)

(ii) The hyperbola of the form $y = \frac{k}{x}$ must be restricted for its inverse to be a function. (1)

(b) The general equation of a hyperbola is $y = \frac{k}{x}$.

Explain what **effect** the constant k, has on the shape of graph of a hyperbola. (2)

(c) The graph of g below represents a semicircle which passes through (1,0) and (-3,0).



(i) Write down the equation of the graph g, sketched above. (4)

(ii) Write down the domain of the inverse of g. (2)

(iii) If the graph of g is reflected about the line $y = -x$, determine algebraically the new equation of g. (3)

(d) On the axis above (on which the graph of g is sketched), sketch the graph of the equation $(x+2)(y-1)=-3$. Call this new graph f. (4)

(e) A circle with equation $x^2 + y^2 + 6x - 4y = k$ has a radius of 4 units.

(i) Calculate the coordinates of the centre of the circle. (3)

(ii) Hence, determine k. (2)

(f) The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$, has the following properties:

- Passes through the point (0, 4).
- Has axis of symmetry $x = -2$.
- Has a maximum value of 10.

Determine a, b and c. (5)