Student's Name [print] Student Number

Mathematics 028b Test 1 **CODE 111** **Wednesday January 26, 2005**

45 minutes

Use the following information for questions 1 and 2.

All of the individual pieces of paper in a certain professor's office can be classified by location: as "Put away in a file or drawer" (set P), as "loose on the Desk" (set D) or as "in a pile on the Floor" (set F).

Alternatively, these same pieces of paper, whether or not they are "put away", can be classified by their importance in any of 3 ways: as "still Important" (set I), as "useful as Scrap paper" (set S) or as "should be Recycled" (set R).

1. Which of the following is the set of all unfiled important pieces of paper which are loose on the desk?

A: $P^c \cup I \cup D$	B: $P^c \cup (I \cap D)$	C: $I \cap D$	D: $I \cup (P^c \cap D)$	E: $(P \cap R)^c$

Solution: The set we want to describe contains pieces of paper which are not filed, i.e. not put away, so they are not in set P. They are important, so they are in set I and they are loose on the desk, so they are also in set D. (Note that $D \subseteq P^c$, i.e. everything which is in D is not in P, so if we specify that the pieces of paper must be in D there is no need to say that they also are in P^c.) The pieces of paper which are in I and also in D are the elements of the set $I \cap D$. Thus the answer is **C**.
Notice that all of the other answer choices include some pieces of paper which are not loose on the desk, for instance some other elements of P^c (which would be in F) and/or some other important pieces of paper which are put away.

2. Which of the following statements about the set $P^c \cap R$ is **not** true?

A: None of these pieces of paper is important.
B: Some of these pages are useful as scrap paper.
C: None of these pieces of paper is in a file.
D: There may be some of these pieces of paper loose on the desk.
E: None of these pages has been put away.

Solution: The elements of the set $P^c \cap R$ are in P^c, i.e. are not in P, and are in R. Thus each of these pieces of paper is *not* put away (in a file or drawer), and *is* a piece of paper which should be recycled, i.e. is *not* important and is *not* useful as scrap. However we don't know anything about the location of these pieces of paper, other than that they are not put away. Some may be loose on the desk and others may be in piles on the floor. Thus all of the answer choices except for **B** are true.

3. There are 100 students in a certain math class. Let M be the set of all male students in the class and P be the set of all students who are passing the class. If there are 57 men in this class and 80% of the students in the class are passing, including 48 of the men, find $n(M \cup P)$.

A: 45.6	B: 89	C: 48	D: 137	E: 91

Solution: We are told that $n(U) = 100$, $n(M) = 57$ and that 80% of the 100 students are in P, i.e., $n(P) = 80$. We are also told that $n(M \cap P) = 48$. We see that

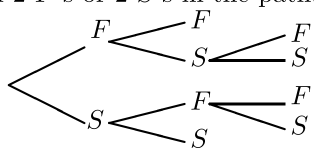
$$n(M \cup P) = n(M) + n(P) - n(M \cap P) = 57 + 80 - 48 = 89$$

The answer is **B**.

4. Fred and Sam are going to play a "best 2 out of 3" match of the card game Cribbage. That is, they will play until one of them has won 2 games. (In Cribbage, there is always a winner; it is not possible for a game to end in a tie.) Considering the number of games played and who wins each game, in how many distinct ways could the Cribbage match turn out? (i.e. how many different sequences of games are possible?)

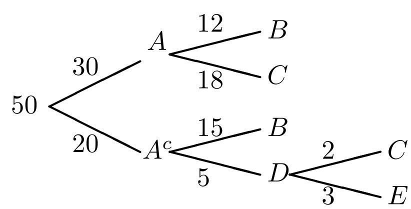
A: 2	B: 4	C: 5	D: 6	E: 8

Solution: We can draw a tree and count the terminal points. We can use F to represent that Fred wins a game and S to indicate that Sam wins. The tree has a level for each game that is played, and a terminal point occurs whenever there have been 2 F's or 2 S's in the path.



We see that there are 6 terminal points in the tree, so the answer is **D**.

5. Consider the counting tree shown below. (Note that $D \subseteq A^c$.) Find $n(C \cap D)$.



A: 2	B: 5	C: 20	D: 21	E: 23

Solution: Since $D \subseteq A^c$, i.e. since D only occurs when A has not occurred, then D does not occur at all in the top half of the tree. The elements of the universal set which are in D and are also in C are those which are represented on the C branch which follows the D branch. There are 2 such elements. The answer is **A**.

6. A small multi-plex movie theatre has 4 different movies showing. All of these movies start at roughly the same time. Three groups of people arrive at the theatre. In how many different ways could these 3 groups each choose a movie to see?

A: $4!$	B: $3!$	C: $\binom{4}{3}$	D: 3^4	E: 4^3

Solution: Each of the 3 groups, separately, will decide "which movie should we see?". For each of these 3 decisions there are 4 choices of movie. Thus, by the fundamental counting principle, there are

$$4 \times 4 \times 4 = 4^3$$

different ways in which the 3 groups could each choose a movie. The answer is **E**.

7. In how many distinct ways can some 4 out of a set of 6 different display plates be arranged (in a row) on a shelf?

A: 15	B: 24	C: 30	D: 360	E: 720

Solution: We need to know the number of permutations of $k = 4$ out of $n = 6$ objects. This is given by

$$\frac{n!}{(n-k)!} = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 6 \times 5 \times 4 \times 3 = 360$$

Alternatively, we could choose 4 plates and then arrange them, which can be done in $\binom{6}{4} \times 4! = \frac{6!}{4!2!} \times 4! = \frac{6!}{2!} = 360$ ways, as above. The answer is **D**.

8. In how many distinct ways can 6 children be arranged in a circle to play a counting game?

A: 5	B: 6	C: 5!	D: 6!	E: 7!

Solution: The number of circular permutations of $n = 6$ children is $(n-1)! = 5!$. The answer is **C**.

9. In how many distinct ways could 3 boys and 4 girls be arranged on a bench so that the girls are all sitting together?

A: 7!	B: 4!	C: $4! \times 3!$	D: $4! \times 3! \times 2$	E: $4! \times 4!$

Solution: We have 3 boys and 4 girls being arranged in a line. The 4 girls must be together. (The boys need not be all together.) There are 3! permutations of the 3 boys. Once the boys are lined up, there are 4 different places in the line that the group of girls could go: before the first boy, between the first and second boys, between the second and third boys, or after the third boy. There are also 4! different permutations of the 4 girls within their group. We see that the number of seating arrangements is given by

$$3! \times 4 \times 4! = 4 \times (3!) \times 4! = (4 \times 3 \times 2 \times 1) \times 4! = 4! \times 4!$$

The answer is **E**.

10. Eugene has 4 mystery novels and 5 science fiction novels which he wants to read. In how many distinct ways can he choose 3 of these books to take with him on vacation next week?

A: $\binom{9}{3}$	B: $\binom{4}{3} + \binom{5}{3}$	C: $\binom{4}{3} \times \binom{5}{3}$	D: $\frac{9!}{3!}$	E: $\frac{9!}{6!}$

Solution: Eugene simply wants to choose any 3 of the $4 + 5 = 9$ books. He can do this in $\binom{9}{3} = \frac{9!}{3!6!}$ different ways. The answer is **A**.

11. Matt has to choose 5 courses for next term. He must choose 3 courses from a list of 6 in his major area of study. He must also take 1 of 3 Math courses. His last course can be any 1 of 10 elective courses. In how many distinct ways can Matt choose courses for next term?

A: $\binom{6}{3} + \binom{3}{1} + \binom{10}{1}$	B: $\frac{6!}{3!} + \frac{3!}{1!} + \frac{10!}{1!}$	C: $6 \times 3 \times 10$	D: $\binom{6}{3} \times \binom{3}{1} \times \binom{10}{1}$	E: $\frac{6!}{3!} \times \frac{3!}{1!} \times \frac{10!}{1!}$

Solution: Matt needs to choose 3 out of the 6 courses on the list, and also choose 1 of the 3 math courses and choose 1 of the 10 electives. Since he must make all 3 of these decisions, the FCP tells us that this can be done in $\binom{6}{3}\binom{3}{1}\binom{10}{1}$ different ways. The answer is **D**.

12. Mary also has to choose 5 courses for next year. The normal requirements for her programme would have her take any 3 of 4 courses in her major area and any 2 from a list of 7 elective courses (2 of which are Economics electives). However, if she decides to declare a minor in Economics, then she only needs to take (any) 2 of the 4 courses in her major area, but must also take a specified Economics course (which is not one of the electives) and must take one of the 2 Economics electives. Her fifth course would be chosen from the list of 5 non-Economics elective courses. In how many distinct ways can Mary choose courses for next year?

A: 39	B: 144	C: 350	D: 1254	E: 5040

Solution: Mary has apparently not yet decided whether she will declare a minor in Economics. This gives 2 cases which must be considered.

Case 1: no minor in Economics
Mary will choose 3 of the 4 courses in her major area and will choose 2 from the 7 electives. The number of distinct ways in which she can do this is

$$\binom{4}{3}\binom{7}{2} = \frac{4!}{3!1!} \times \frac{7!}{2!5!} = 4 \times \frac{7 \times 6}{2} = 84$$

Case 2: Mary declares a minor in Economics
In this case Mary will choose 2 of the 4 courses in her major area, must choose the 1 specified Economics course and choose 1 of the 2 Economics electives, and finally will choose 1 of the 5 other electives. The number of ways in which she can do this is

$$\binom{4}{2}\binom{1}{1}\binom{2}{1}\binom{5}{1} = \frac{4!}{2!2!} \times 1 \times 2 \times 5 = \frac{4 \times 3}{2} \times 10 = 60$$

Altogether, there are a total of $84 + 60 = 144$ distinct ways in which Mary could choose courses for next year. (That is, since she will do one *or* the other, we add.) The answer is **B**.

13. In how many distinct ways can the letters of the word BABBOON be arranged?

A: $7!$	B: $3!2!1!1!4!$	C: $\binom{7}{3\ 2\ 2}$	D: $\binom{7}{3\ 2\ 1\ 1}$	E: $\binom{7}{3}\binom{7}{2}\binom{7}{1}\binom{7}{1}$

Solution: This is a straightforward labelling problem, labelling the position of the 7-letter word using 3 B labels, 2 O labels, 1 A label and 1 N label. The number of permutations is $\binom{7}{3\ 2\ 1\ 1}$ so the answer is **D**.

14. Claire is on vacation in New Zealand. She has purchased 8 different postcards and will send 2 to each of 4 different people. Without considering the order in which she sends them, in how many different ways can she decide *which* 2 postcards to send to each person?

A: $\binom{8}{2\ 2\ 2\ 2}$	B: $\binom{8}{2\ 2\ 2\ 2} \div 4!$	C: $\binom{8}{4\ 4}$	D: $\binom{8}{4\ 4} \div 2$	E: $8! \div 4!$

Solution: Claire has 8 distinct postcards and wants to divide them into 4 groups of 2. The groups are distinguishable because each group will be sent to a different one of Claire's friends or relatives. The number of ways in which Claire can divide the 8 postcards into these 4 groups is $\binom{8}{2\ 2\ 2\ 2}$. The answer is **A**.

15. Claire has just returned from her New Zealand vacation. She bought herself 8 small statuettes of Maori (New Zealand native people), each in a different position. She wants to display them in 2 groups of 4. In how many distinct ways can she divide the statuettes into display groups?

A: $\binom{8}{2\ 2\ 2\ 2}$	B: $\binom{8}{2\ 2\ 2\ 2} \div 4!$	C: $\binom{8}{4\ 4}$	D: $\binom{8}{4\ 4} \div 2$	E: $8! \div 4!$

Solution: Since each of the statuettes is in a different position, there are 8 distinct statuettes. Claire wishes to divide these into 2 groups of 4 each. The groups are indistinguishable because each will be doing the same thing – 'being displayed in a group together'. The number of ways in which 8 distinct objects can be divided into 2 indistinguishable groups of 4 is $\binom{8}{4\ 4} \div 2! = \binom{8}{4\ 4} \div 2$. The answer is **D**.