1. In a poll, it was discovered that all of the students in a certain High School class have a VCR, a DVD player or both in their home. When asked, 30 students answered yes to "Is there a VCR in your home?", and 20 students answered yes to "Is there a DVD player in your home?". If 14 of the students answered yes to both questions, how many students are in the class?

A: 24	B: 30	C: 36	D: 64	E: Cannot be determined.

2. A certain office building contains 100 offices. There are 20 large offices, 35 medium-sized offices and 45 small offices. All of the large offices have windows, but only 15 of the small offices have windows. If 60 of the offices in the building have windows, how many of the medium-sized offices do not have a window?

A: 0	B: 10	C: 25	D: 30	E: Cannot be determined.

3. How many (positive) even integers less than 5000 can be formed using only digits which are in the set $\{1, 2, 3, 4\}$, if repetition is allowed?

A: 12	B: 32	C: 128	D: 170	E: 340

4. Each student in a certain class must write an essay. The professor has provided a list of 6 acceptable essay topics. (Each student must choose one of these topics for his or her essay.) If there are 15 students in the class, in how many different ways can the students choose topics for their essays?

A: $15!$	B: $\frac{15!}{9!}$	C: $\binom{15}{6}$	D: 15^6	E: 6^{15}

5. Ted is having dinner at a restaurant and has ordered ice cream for dessert. The waitress says there are 5 flavours of ice cream available. As well, she asks whether Ted would like any one topping (strawberries, chocolate sauce or butterscotch), and/or whipped cream on his ice cream. Finally, she asks whether he wants chocolate sprinkles. In how many different ways could Ted order his ice cream?

A: 13	B: 15	C: 32	D: 60	E: 80

6. Using the letters $a, b, c, d, e, f, g, h, i$ and j, how many 3-letter passwords can be formed if repetition is not allowed?

A: 1000	B: 729	C: 720	D: 504	E: 120

7. Refer to question 6. How many 3-letter passwords can be formed if repetition *is* allowed?

A: 1000	B: 729	C: 720	D: 504	E: 120

8. How many subsets of the digits 0 through 9 are there?

A: 2^9	B: 2^{10}	C: 9^2	D: 10^2	E: $10!$

9. How many subsets of 3 of the digits 0 through 9 are there?

A: 1000	B: 729	C: 720	D: 504	E: 120

10. In how many different ways can the letters of the word 'oranges' be arranged?

A: 7	B: $7!$	C: 2^7	D: 7^7	E: $\binom{26}{7}$

11. In how many different ways can the letters of the word 'bananas' be arranged?

A: $7!$	B: 2^7	C: $\binom{7}{3} \times \binom{7}{2}$	D: $\frac{7!}{3!2!}$	E: $\frac{7!}{3!2!} \times \frac{1}{2!}$

12. Terry is packing lunches for 6 different children. There are 3 apples, 2 bananas and 1 orange. In how many distinct ways can Terry put 1 piece of fruit in each child's lunch? (Assume fruit of the same type are indistinguishable, so only the type of fruit matters.)

A: 12	B: 24	C: 41	D: 60	E: 1800

13. There are 6 different books which Gary and Mary must read to research a project they're working on together. In how many different ways can they divide up the books between them if each is going to read 3 books?

A: $\binom{6}{3\ 3}$	B: $\binom{6}{3\ 3} \div 2!$	C: $\binom{6}{3} + \binom{3}{3}$	D: 2^6	E: $6!$

14. There are 6 different books to be placed in 2 identical carrier bags. In how many distinct ways can the books be divided up to be put in the bags if 3 books are to be put in each bag?

A: $\binom{6}{3\ 3}$	B: $\binom{6}{3\ 3} \div 2!$	C: $\binom{7}{2}$	D: $\binom{7}{6}$	E: 2^6

15. There are 6 identical copies of the same book to be placed in 2 identical carrier bags. In how many distinct ways can the books be divided up to be put in the bags if 3 books are to be put in each bag?

A: $\binom{6}{3\ 3}$	B: $\binom{6}{3\ 3} \div 2!$	C: $\binom{7}{6}$	D: 2^6	E: 1

16. The London Public Library has just received 40 (identical) copies of a recent best selling book. In how many distinct ways can these books be distributed among the 15 different library branches if each branch must receive at least 1 copy of the book?

A: $\binom{54}{40}$	B: $\binom{54}{15}$	C: $\binom{39}{25}$	D: $\binom{39}{15}$	E: 15^{40}

17. Every customer who orders an extra large pizza today from Pepe's Perfect Pizza will be offered the anniversary special: they can either get a small pizza (along with their extra large) for an extra \$2, or they can get a second extra large pizza for an extra \$5. They cannot get both, and some customers, of course, may not want either. Pepe wants to know how many different possibilities there are for having to provide these special pizzas, assuming that 50 people order extra large pizzas today (i.e. 50 people are offered this special). Pepe does not care about which customers order the small or extra large special. He just cares about how many customers order each. (e.g. 50 take the small special, or 20 take the small special, 20 take the XL special and 10 take neither.) How many different possibilities are there?

A: 3^{50}	B: 50^3	C: $\binom{52}{50}$	D: $\binom{52}{3}$	E: 50×3

18. $S = \{a, b, c\}$ is a sample space for a certain experiment. If $\Pr[c] = 2\Pr[b]$ and $\Pr[b] = 3\Pr[a]$, what is the value of $\Pr[a]$?

A: $\frac{1}{10}$	B: $\frac{1}{6}$	C: $\frac{1}{3}$	D: $\frac{6}{10}$	E: Cannot be determined.

19. Six children and one teacher sit in a circle to play a game. If they arrange themselves at random in the circle, what is the probability that Mary is sitting beside the teacher?

A: $\frac{1}{6}$	B: $\frac{1}{3}$	C: $\frac{2}{7}$	D: $\frac{2}{6!}$	E: $\frac{1}{15}$

20. A 3 card hand is dealt at random from a standard deck of cards. What is the probability that the hand contains at least 2 clubs?

A: $\frac{\binom{13}{2}\binom{39}{1}+\binom{13}{3}}{\binom{52}{3}}$	B: $\frac{\binom{13}{2}+\binom{13}{3}}{\binom{52}{3}}$	C: $\frac{\binom{13}{2}\binom{39}{1}}{\binom{52}{3}}$	D: $\frac{\binom{13}{2}\binom{50}{1}}{\binom{52}{3}}$	E: $\frac{13 \times 13 \times 39}{(52)^3}$

21. A 3 card hand is dealt at random from a standard deck of cards. If it is known that the hand contains *exactly* 2 clubs, what is the probability that the hand contains exactly 1 diamond?

A: 0	B: $\frac{1}{4}$	C: approx. .3047	D: $\frac{1}{3}$	E: 1

22. A single card is drawn from a standard deck. Let H be the event that a Heart is drawn and E be the event that the card is not a spade. Find $\Pr[H \mid E]$.

A: $\frac{\binom{13}{1}\binom{39}{1}}{\binom{52}{2}}$	B: $\frac{3}{4}$	C: $\frac{1}{2}$	D: $\frac{1}{4}$	E: $\frac{1}{3}$

23. Refer to question 22. Are H and E independent events?

A: Yes	B: No	C: Cannot be determined.

24. There are 3 identical-looking boxes. One box contains a triangle, a square and a circle. The second box contains a triangle and an oval. The third box contains one square and 2 triangles. One box is selected at random and a single shape is chosen from that box. What is the probability that the chosen shape is an oval?

A: $\frac{1}{8}$	B: $\frac{1}{6}$	C: $\frac{1}{4}$	D: $\frac{1}{3}$	E: $\frac{1}{2}$

25. In a large class, 60% of the students are women. It is known that 70% of the women and 30% of the men in this class have long (at least shoulder length) hair. A single student is chosen at random from the class. What is the probability that the student has long hair?

A: .3	B: .42	C: .54	D: .7	E: 1

26. Refer to question 25. If the chosen student does have long hair, what is the probability that the student is a man?

A: $\frac{2}{9}$	B: $\frac{2}{7}$	C: $\frac{3}{25}$	D: $\frac{3}{10}$	E: $\frac{27}{125}$

Use the following information for questions 27, 28 and 29.

10 standard decks of cards are shuffled (separately) and a single card is drawn from each deck.

27. What is the probability that exactly 1 Heart is drawn?

A: $\left(\frac{1}{4}\right)^{10}$	B: $\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{9}$	C: $10 \times \left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{9}$	D: $\frac{\binom{520}{130}}{\binom{520}{10}}$	E: $\frac{\binom{130}{1}\binom{390}{9}}{\binom{520}{10}}$

28. What is the probability that at least 1 Heart is drawn?

A: $10 \times \left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{9}$	B: $1 - 10 \times \left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{9}$	C: $\left(\frac{3}{4}\right)^{10}$
D: $1 - \left(\frac{3}{4}\right)^{10}$	E: $\left(\frac{1}{4}\right)^{1+2+\cdots+10}$	

29. (Refer to previous information.) What is the probability that 2 Hearts, 3 Spades, 4 Clubs and 1 Diamond are drawn?

A: $\left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)^3 \left(\frac{1}{4}\right)^4 \left(\frac{1}{4}\right)^1$
B: $\frac{10!}{2!3!4!1!} \left(\frac{1}{4}\right)^{10}$
C: $\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^4 + \left(\frac{1}{4}\right)^1$
D: $\binom{10}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^8 + \binom{10}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^7 + \binom{10}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^6 + \binom{10}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^9$
E: $10! \times \left(\frac{1}{4}\right)^{10}$

30. I have 6 loose keys in a box. Two of these keys are spare keys for my front door. The others are keys to neighbours' and friends' houses. The keys all look similar, so I can't tell which ones will unlock my front door. I need to find a spare key for my house, so I choose keys randomly from the box (without replacement) and try them in my front door until I find one that works. Let X be the number of keys left in the box when I find one that works. What is $\Pr[X \geq 3]$?

A: $\frac{1}{5}$	B: $\frac{2}{5}$	C: $\frac{3}{5}$	D: $\frac{4}{5}$	E: 1

31. Let X be a discrete random variable with $\Pr[X = -1] = \frac{1}{3}, \Pr[X = 0] = \frac{1}{6}$ and $\Pr[X = 1] = \frac{1}{2}$. What is the mean of X?

A: 0	B: $\frac{1}{36}$	C: $\frac{1}{6}$	D: $\frac{5}{6}$	E: Cannot be determined.

32. Discrete random variable X has $\Pr[X = 1] = \frac{1}{4}$ and $\Pr[X = 2] = \frac{3}{4}$, so that $\mu = \frac{7}{4}$. Find $V(X)$.

A: $\frac{7}{4}$	B: $\frac{49}{16}$	C: $\frac{13}{4}$	D: $\frac{3}{16}$	E: $\frac{\sqrt{3}}{4}$

33. Let X be a discrete random variable with mean $\mu = 5$ and variance $V(X) = 2$. What is the standard deviation of X?

A: 27	B: -21	C: 2	D: $\sqrt{21}$	E: $\sqrt{2}$

34. Let X be a discrete random variable with mean $\mu = 8$ and variance $V(X) = 36$. What is the value of $E(X^2)$?

A: 100	B: 64	C: 44	D: 14	E: 6

Use the following information for questions 35, 36 and 37.

Discrete random variable X has $E(X) = 10$ and $V(X) = 4$. Y is a discrete random variable whose value is given by $Y = 25 - 2X$.

35. Find $E(Y)$.

A: 20	B: -20	C: 5	D: 17	E: Cannot be determined.

36. Find $\sigma(Y)$.

A: 4	B: -4	C: 5	D: 16	E: 21

37. Are X and Y independent random variables?

A: Yes	B: No	C: Cannot be determined.

38. A fair coin is tossed twice. Discrete random variable X has the value 0 if the first toss comes up Heads and otherwise has the value 1. Discrete random variable Y is the number of times the coin comes up Heads during the 2 tosses. Find $V(X + Y)$.

A: $\frac{1}{4}$	B: $\frac{1}{2}$	C: $\frac{3}{4}$	D: $\frac{3}{2}$	E: $\frac{5}{2}$

39. A fair coin is tossed 10 times. Let X be the number of times Tails comes up. Which of the following statements is **false**?

A: $E(X) = 5$	B: $V(X) = \frac{5}{2}$
C: $\Pr[X = 5] = \frac{63}{256}$	D: $\Pr[X \geq 1] = 1 - \frac{1}{2^{10}}$
E: The possible values of X are the integers from 1 to 10.	

Use the following information for questions 40 and 41.

A single card is drawn from a well-shuffled deck. Discrete random variable X has the value 1 if the card is black and 3 if the card is red. Discrete random variable Y has the value 1 when the card is a 2,3,4,5,6,7,8,9 or 10 and has the value 5 when the card is a Jack, Queen, King or Ace. Values of $\Pr[(X = x) \cap (Y = y)]$ are shown in the table below:

	$(Y = 1)$	$(Y = 5)$
$(X = 1)$	$18/52$	$8/52$
$(X = 3)$	$18/52$	$8/52$

It can easily be verified that $E(X) = 2$ and $\sigma(X) = 1$, while $E(Y) = \dfrac{29}{13}$ and $\sigma(Y) = \dfrac{24}{13}$.

40. Are X and Y independent random variables?

A: Yes	B: No	C: Cannot be determined.

41. (Refer to above.) You pay \$5 to play a game in which you will draw a card and receive \$W, where $W = XY$. What is the expected value of your net winnings from playing this game? (Answer to the nearest cent.)

A: \$9.46	B: \$4.46	C: \$1.85	D: -\$0.54	E: -\$1.59

42. Continuous random variable X has probabilitiy density function $f(x) = \dfrac{2x}{25}$ if $0 \le x \le 5$ and $f(x) = 0$ otherwise. Find $\Pr[X < 3]$.

A: $\frac{4}{25}$	B: $\frac{6}{25}$	C: $\frac{9}{25}$	D: $\frac{16}{25}$	E: $\frac{1}{4}$

43. Continuous random variable X has probability density function $f(x) = cx$ if $0 \le x \le 3$ and $f(x) = 0$ otherwise. What is the value of c?

A: $\frac{9}{2}$	B: 3	C: $\frac{1}{3}$	D: $\frac{2}{9}$	E: Cannot be determined.

44. Discrete random variable X has possible values 2,4,6 and 8. Continuous random variable Y is a good approximation for X. Which of the following approximates $\Pr[X \le 6]$?

A: $\Pr[5.5 < Y < 6.5]$	B: $\Pr[5 < Y < 7]$	C: $\Pr[Y < 6]$
D: $\Pr[Y < 6.5]$	E: $\Pr[Y < 7]$	

45. Discrete random variable X has as its possible values the integers 1 through 10. Continuous random variable Y is a good approximation for X. Which of the following approximates $\Pr[3 < X < 8]$?

A: $\Pr[3 < Y < 8]$	B: $\Pr[3.5 < Y < 7.5]$	C: $\Pr[4 < Y < 7]$
D: $\Pr[2.5 < Y < 8.5]$	E: $\Pr[Y < 7.5] - \Pr[Y < 2.5]$	

46. X is a discrete random variable whose possible values are consecutive integers. Continuous random variable Y has probability density function $f(y) = \frac{1}{6}$ if $2.5 \le y \le 8.5$ and $f(y) = 0$ otherwise. Y is a good approximation for X. Which of the following statements is **false**?

A: The possible values of X are equally likely to occur.
B: The largest possible value of X is 8.
C: X has 6 possible values.
D: $\Pr[X > 3] = \frac{5}{6}$
E: $\Pr[3 < X < 5] = \frac{1}{2}$

In questions 47 through 50, Z is the Standard Normal random variable. The table showing values of the cumulative distribution function for Z is given at the end of the exam.

47. Find $\Pr[Z < 1.75]$.

A: 0.401	B: .0409	C: .9591	D: .9599	E: 1.75

48. Find $\Pr[Z < -0.45]$.

A: .1736	B: .3264	C: .45	D: .55	E: .6736

49. Find $\Pr[0.40 < Z < 1.00]$.

A: .60	B: .4967	C: .1859	D: .1554	E: $-.1859$

50. Find $\Pr[-1.2 < Z < 0.7]$.

A: .9000	B: .8731	C: .6915	D: .6429	E: .1269

51. X is a normal random variable with mean $\mu = 50$ and standard deviation $\sigma = 4$. Find $\Pr[X < 55]$.

A: .6217	B: .8708	C: .8944	D: .9162	E: .9938

52. X is a normal random variable with mean $\mu = 35$ and standard deviation $\sigma = 10$. Find $\Pr[20 < X < 50]$.

A: .8664	B: .8530	C: .8788	D: .1192	E: .999997

53. The actual weight of a "1 pound" block of butter is normally distributed with mean μ and standard deviation $\sigma = 2$ grams. To the nearest gram, what is the minimum number of grams that the mean weight, μ, should be in order to ensure that there is less than a 1% chance that a "1 pound" block of butter actually contains less than 1 pound, i.e. 454 grams, of butter?

A: 455	B: 456	C: 457	D: 458	E: 459

54. The number of paper clips in a box is approximately normally distributed, with mean 100 and standard deviation 3. What is the probability that a box contains fewer than 102 paper clips?

A: .6915	B: .7486	C: .7967	D: .8078	E: .5000

55. According to a recent poll, 36% of the population object to same sex marriage. If 100 people are selected at random, what is the (approximate) probability that no more than 39 of them object to same sex marriage?

A: .5596	B: .6628	C: .6985	D: .7537	E: .7673

The Cumulative Distribution Table for the Standard Normal Random Variable Z

$Pr[Z < k]$

k	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

Student Number

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 028b Final Examination

Friday, April 23, 2004 Code 111 7:00 p.m. - 10:00 p.m.

INSTRUCTIONS

1. The examination consists of 20 multiple choice questions.

2. Calculators are allowed. No other aids are permitted.

3. Questions start on Page 1 and continue to Page 4. Be sure that your booklet is complete. Questions are printed on both sides of the paper. Scrap paper will be provided for rough work.

4. **Circle** the correct answer to each question **on this paper** and then fill in the appropriate box on the **scantron** card with an HB pencil. Fill in the top of the scantron card with your Name and Student Number (both printing and coding). You must hand in this question paper, your scantron card, as well as any rough work sheets.

5. TOTAL MARKS = 20.