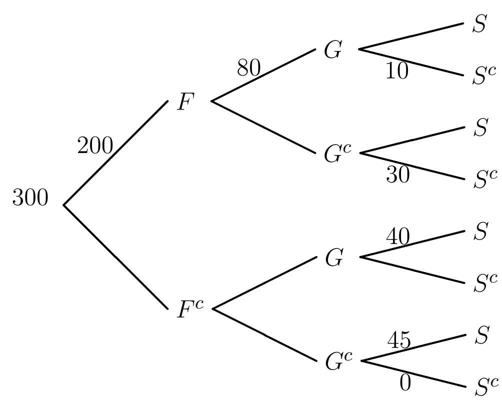
1. Let E and F be subsets of a universal set U with $n(U) = 100$, $n(E) = 40$, $n(E \cap F^c) = 30$ and $n(E^c \cap F) = 20$. Find $n(E \cap F)$.

A: 50	B: 40	C: 20	D: 10	E: 5

2. A fair coin is tossed at most four times. The tossing stops as soon as two heads in a row appear, two tails in a row appear, or four tosses have been made. How many sequences of tosses are possible? Use a tree diagram.

A: 2	B: 6	C: 8	D: 10	E: 16

3. A survey of 300 tourists returning to Canada asked tourists whether they had visited France (F), Germany (G), or Spain (S). Some of the numbers from the survey are displayed on the counting tree below.



How many tourists visited Germany?

A: 135	B: 80	C: 55	D: 40	E: 10

4. See question 3. Find $n(F^c \cup S)$.

A: 235	B: 260	C: 165	D: 145	E: 100

5. How many five digit numbers more than 35000 can be made using the digits 2,3,4,5,6,7,8 where each digit may be used as many times as desired?

A: $5^4 + 6^5$	B: $3 \times 7^3 + 4 \times 7^4$	C: 5×7^4	D: $5(7^3 + 7^4)$	E: $4 \times 7^3 + 5 \times 7^4$

6. In how many ways can three girls and six boys arrange themselves in single file at an ice-cream stand if the three girls stay together in line?

A: $3!6! \times 2$	B: $7!$	C: $3!6!$	D: $3!7!$	E: $9!$

7. Ace Hardware has hired 10 clerks to staff its two stores. In how many ways can the 10 clerks be assigned to the two stores if the only restriction is that each store must receive at least one clerk?

A: 2^8	B: 2^{10}	C: $2^{10} - 2$	D: $2^{10} - 1$	E: $10! - 2$

8. Eight people are to be seated around a large round table for a conference. Find the number of possible seating arrangements.

A: $8!$	B: $7!$	C: 8	D: 7	E: 2^8

9. Eight students entering a cafeteria for lunch notice that the only seats available are 5 chairs at an empty round table and 3 chairs along the wall across the room from the table. In how many ways can the eight students arrange themselves in the eight chairs?

A: $\binom{8}{5}$	B: $\binom{8}{5}4!2!$	C: $\binom{8}{5}4!3!$	D: $4!3!$	E: $8!$

10. A committee consisting of 5 members is to be selected from among 9 women and 7 men. In how many ways can this be done if the committee must contain at least one man?

A: $\binom{16}{5} - \binom{7}{5}$	B: $\binom{16}{5} - \binom{9}{5}$	C: $\binom{16}{5} - 1$	D: $\binom{16}{5} - \binom{9}{5}\binom{7}{5}$	E: $\binom{9}{5}$

11. At a dance there are 10 women and 20 men without dance partners. In how many ways can the 10 women be paired with 10 of the men for the next dance?

A: $\binom{20}{10}$	B: $\binom{20}{10}(10!)^2$	C: $2^{10}10!$	D: $10 \times \binom{10}{1}\binom{20}{1}$	E: $\binom{20}{10}10!$

12. In how many ways can 12 Business students be divided into three groups of 4 students to gain teamwork experience? Assume that each group is engaged in the same activity.

A: $\frac{1}{3!}\binom{12}{4\ 4\ 4}$	B: $\frac{1}{3}\binom{12}{4\ 4\ 4}$	C: $3!\binom{12}{4\ 4\ 4}$	D: $\binom{12}{4}^3$	E: $\binom{12}{4\ 4\ 4}$

13. See question 12. In how many ways can the 12 students be divided into the three groups of 4 if Charles and Henry (two of the twelve students) must not belong to the same group?

A: $\frac{1}{2!}\binom{10}{3\ 3\ 4}$	B: $\binom{10}{3\ 3\ 4}$	C: $2!\binom{10}{3\ 3\ 4}$	D: $\binom{12}{2}\binom{10}{3\ 3\ 4}$	E: $\binom{12}{4\ 4\ 4} - \binom{12}{2}$

14. How many distinct 10 digit numbers can be formed by permuting (i.e. arranging) the digits 1,1,1,2,2,2,3,3,4,5?

A: $10!$	B: $\binom{10}{3}\binom{7}{3}\binom{4}{2}\binom{2}{1}4!$	C: $\frac{1}{2!}\frac{1}{2!}\binom{10}{3\ 3\ 2\ 1\ 1}$
D: $\binom{10}{3\ 3\ 2\ 2}$	E: $\binom{10}{3\ 3\ 2\ 1\ 1}$	

15. Two cards are selected at random (without replacement) from a standard deck of 52 cards. What is the probability that neither card is a heart?

A: $\frac{19}{34}$	B: $\frac{9}{16}$	C: $\frac{3}{4}$	D: $1-\frac{2\times 13}{52}$	E: $\frac{3}{17}$

16. Six customers interested in purchasing computers each randomly choose to deal with either salesperson A or salesperson B. Find the probability that exactly three customers decide to deal with salesperson A.

A: $\frac{3!\binom{6}{3}}{2^6}$	B: $\frac{1}{2}$	C: $\frac{\binom{6}{3}}{2^6}$	D: $\frac{\binom{6}{3}}{6!}$	E: $\frac{6\times 5\times 4}{6!}$

17. Find the probability that the letters A, B and C appear all together in a random permutation of the seven letters A, B, C, D, E, F and G.

A: $\frac{3}{7}$	B: $\frac{4!}{7!}$	C: $\frac{\binom{7}{3}4!}{2^7}$	D: $\frac{4!3!}{7!}$	E: $\frac{5!3!}{7!}$

18. On a final exam a student is asked to answer 5 out of 10 questions. Haley knows the answer to all 10 questions and randomly selects 5 questions to answer. Find the probability that his selection includes at least one of the first three questions.

A: $\frac{7}{10}$	B: $\frac{\binom{7}{5}}{\binom{10}{5}}$	C: $1-\frac{\binom{7}{5}}{\binom{10}{5}}$	D: $1-\frac{5!\binom{7}{5}}{\binom{10}{5}}$	E: $1-\frac{\binom{7}{5}}{10!}$

19. Events E and F in a sample space S have $\Pr[E]=\frac{1}{2}$, $\Pr[F]=\frac{3}{4}$ and $\Pr[E\mid F]=\frac{1}{3}$. Find $\Pr[E\cap F]$.

A: $\frac{1}{2}$	B: $\frac{1}{4}$	C: $\frac{3}{4}$	D: $\frac{1}{8}$	E: $\frac{7}{8}$

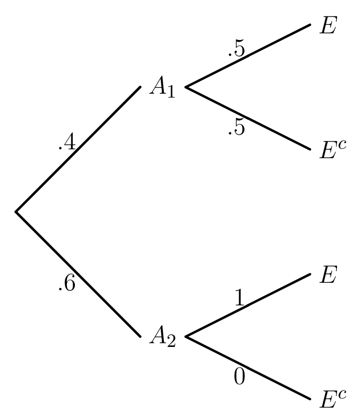
20. Three fair dice are tossed and the sum of the numbers which show is observed. If it is known that the same number came up on all of the dice, what is the probability that the sum is 18?

A: $\frac{1}{36}$	B: $\frac{1}{6}$	C: $\frac{1}{6^3}$	D: $\frac{5}{6}$	E: $\frac{18}{37}$

21. Let E and F be events with $\Pr[E] = \frac{5}{8}$, $\Pr[F] = \frac{2}{5}$ and $\Pr[E \cap F] = \frac{1}{4}$. Are E and F independent events?

A: Yes	B: No	C: Cannot be determined

The following probability tree applies to questions 22, 23 and 24.



22. Find $\Pr[E]$.

A: 0.4	B: 1	C: 0.6	D: 0.5	E: 0.8

23. Find $\Pr[E \mid A_1]$.

A: 0.4	B: 0.3	C: 0.6	D: 0.5	E: 1

24. Find $\Pr[E \cap A_1]$.

A: 0.2	B: 0.5	C: 0.6	D: 0.9	E: 0.4

25. On any given day, there is an 80% chance that an absent minded professor does research and a 20% chance that he cleans his office. When he does research he always forgets his keys when he leaves his office, but when he cleans his office he only forgets his keys half the time. Last Tuesday, the professor forgot his keys when he left his office. What is the probability that he did research on that day?

A: $\frac{5}{9}$	B: $\frac{8}{9}$	C: $\frac{1}{2}$	D: $\frac{7}{9}$	E: $\frac{8}{63}$

26. See question 25. What is the probability that the professor will remember his keys when he leaves his office next Tuesday?

A: $\frac{1}{2}$	B: $\frac{2}{9}$	C: $\frac{1}{10}$	D: $\frac{2}{10}$	E: $\frac{3}{9}$

27. Let E be the event that a 5 or a 6 shows on a single toss of a die. What is the probability that E occurs at least once in 3 tosses?

A: $\frac{2}{9}$	B: $\frac{1}{3}$	C: $\frac{26}{27}$	D: $\frac{1}{27}$	E: $\frac{19}{27}$

28. George is not a very good computer technician. Each time that George tries to install a computer, there is only a 50% chance that the installation is completely successful. 30% of the time the installation is only partly successful and 20% of the time the computer is damaged beyond repair. George has been assigned 10 computers to install. What is the probability that exactly 7 computers are damaged beyond repair and only one installation is completely successful?

A: $\binom{10}{7\ 2\ 1}(.2)^2(.3)^7(.5)$	B: $\binom{10}{7\ 2\ 1}(.2)^7(.3)^2(.5)$	C: $\binom{10}{5\ 3\ 2}(.2)(.3)^2(.5)^7$
D: $\binom{10}{5\ 3\ 2}(.2)^2(.3)^7(.5)$	E: $\binom{10}{7\ 3}(.2)^7(.5)^3$	

29. See question 28. What is the probability that all 10 computers are damaged beyond repair?

A: $\left(\frac{7}{10}\right)^{10}$	B: $(.5)^{20}$	C: $\binom{10}{5\ 3\ 2}(.2)^{10}$	D: $\binom{10}{7\ 2\ 1}(.5)(.3)(.2)$	E: $(.2)^{10}$

30. See question 28. What is the probability that at least one computer is damaged beyond repair?

A: $1-(.2)^{10}$	B: $(.2)^{10}$	C: $(.5)^{10}$	D: $1-(.8)^{10}$	E: $1-(.5)^{10}$

31. A 3-card hand is dealt from a standard deck of cards. The discrete random variable X counts the number of clubs in the hand. Considering the sample space to be the set of all possible 3-card hands which could be dealt, so that $n(s) = \binom{52}{3}$, how many sample points are in the event $(X = 1)$?

A: 13	B: $\binom{13}{1}\binom{39}{2}$	C: $\binom{13}{1}\binom{39}{2}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2$	D: $\binom{39}{2}$	E: 4

The discrete random variable X has the probability distribution function shown below. Use this information for questions 32 and 33.

x	$\Pr[X = x]$
0	2/9
1	4/9
2	1/3

32. Find $\Pr[X > 0]$.

A: $\frac{7}{9}$	B: $\frac{2}{9}$	C: $\frac{5}{9}$	D: 1	E: 0

33. Find $E(X)$.

A: 1	B: $\frac{4}{3}$	C: $\frac{10}{9}$	D: $\frac{1}{3}$	E: $\frac{44}{81}$

34. The discrete random variable X has possible values $x = 4, 5$ and 6. The cumulative distribution function for X is shown in the table below. Find $\Pr[X = 5]$.

x	$F(x) = \Pr[X \leq x]$
4	1/4
5	2/3
6	1

A: $\frac{2}{3}$	B: $\frac{11}{12}$	C: $\frac{1}{5}$	D: $\frac{7}{12}$	E: $\frac{5}{12}$

35. A bag contains 2 red disks and 3 green disks. Two disks are drawn from the bag, without replacement. The discrete random variable X counts the number of red disks drawn. What is the value of $\Pr[X = 1]$?

A: 0.3	B: 0.4	C: 0.5	D: 0.6	E: 1.0

36. X is a discrete random variable with $\Pr[X = 0] = \frac{1}{5}$ and $\Pr[X = 1] = \frac{4}{5}$. What is the value of $\sigma(X)$?

A: 1	B: $\frac{4}{5}$	C: $\frac{4}{25}$	D: $\frac{2}{5}$	E: $\frac{3}{5}$

37. X is a discrete random variable with $\Pr[X = -2] = \frac{1}{2}$ and $\Pr[X = 1] = \frac{1}{2}$. What is the variance of X?

A: $-\frac{1}{2}$	B: $\frac{5}{2}$	C: $\frac{9}{4}$	D: $\frac{11}{4}$	E: $\frac{1}{4}$

A certain experiment has possible outcomes t_1, t_2 and t_3. Discrete random variables X and Y are defined on the sample space $\{t_1, t_2, t_3\}$ as shown in the table below. Questions 38, 39 and 40 refer to these random variables.

outcome	probability	value of X	value of Y
t_1	.5	0	5
t_2	.3	1	10
t_3	.2	2	20

38. What is the value of $\Pr[(X = 1) \cap (Y = 5)]$?

A: 0	B: .8	C: .15	D: .5	E: .3

39. What is the value of $\Pr[XY = 0]$?

A: 0	B: .5	C: 11	D: .7	E: 2.38

40. Find $E(XY)$.

A: 0	B: .5	C: 11	D: .7	E: .238

41. X is a discrete random variable with $E(X) = 3$ and $V(X) = 1$. Y is a discrete random variable with $E(Y) = -2$ and $V(Y) = 4$. Find $E(5X - 3Y)$.

A: -7	B: 17	C: 0	D: 9	E: 21

42. If $Y = 2X - 1$ and $V(X) = 9$, what is $\sigma(Y)$?

A: 36	B: $\sqrt{18}$	C: $\sqrt{35}$	D: 6	E: 5

43. The discrete random variable X is defined to be the number of successes observed in n Bernoulli trials with probability of success p. Which of the following statements is **false**?

A: The value of X must be a non-negative integer.
B: X has a binomial distribution.
C: The mean of X is $\mu = np$.
D: The variance of X is $V(X) = np(1-p)$.
E: X is a normal random variable.

44. Let X and Y be two discrete random variables with $\Pr[(X=0)\cap(Y=1)] = .2$, $\Pr[(X=0)\cap(Y=2)] = .1$, $\Pr[(X=1)\cap(Y=1)] = .4$ and $\Pr[(X=1)\cap(Y=2) = .3$. Which of the following statements is false?

A: $\Pr[X=0] = .3$
B: $\Pr[Y=1] = .6$
C: X and Y are independent random variables.
D: The only possible values of X are 0 and 1.

45. X is a discrete random variable with $E(X) = 1$ and $V(X) = \frac{1}{4}$. Y is a discrete random variable with $E(Y) = -1$ and $V(Y) = \frac{1}{25}$. If X and Y are independent random variables, what is the value of $\sigma(X+Y)$?

A: 0	B: 0.29	C: $\sqrt{0.29}$	D: 0.7	E: $\sqrt{0.21}$

46. The continuous random variable X has density function $f(x)$ defined by: $f(x) = \frac{x}{18}$ if $0 \le X \le 6$ and $f(x) = 0$ otherwise. Find $\Pr[X < 3]$.

A: $\frac{1}{2}$	B: $\frac{1}{4}$	C: $\frac{1}{6}$	D: $\frac{1}{36}$	E: 0

47. X is a discrete random variable with possible values $x = 2, 4, 6$ and 8. X has been approximated by a continuous random variable Y. Which of the following is an approximation for $\Pr[X \le 4]$?

A: $\Pr[3 < Y < 5]$	B: $\Pr[Y \le 4]$	C: $\Pr[Y < 4.5]$	D: $\Pr[Y < 5]$	E: $\Pr[Y > 3]$

48. The discrete random variable X has possible values $x = 0, 1, 2, 3, 4$ and 5. X has been approximated by a continuous random variable Y. Which of the following is approximated by $\Pr[.5 < Y < 1.5]$?

A: $\Pr[X = 1]$	B: $\Pr[0 \le X \le 1]$	C: $\Pr[X < 1.5]$
D: $\Pr[X \le 2]$	E: $\Pr[0 < X \le 2]$	

49. For the standard normal random variable Z, what is the value of $\Pr[Z > 1.54]$?

A: .9382	B: .0618	C: .9370	D: .0630	E: .0668

50. If Z is the standard normal random variable, find $\Pr[-1.2 < Z < 1.2]$.

A: .7698	B: .8849	C: .2302	D: 0	E: 1.7698

51. Find the value of k if it is known that $\Pr[k < Z < 1.3] = .2853$, where Z is the standard normal random variable.

A: .2853	B: .7147	C: .57	D: .6179	E: .30

52. X is a normal random variable with mean $\mu = 100$ and standard deviation $\sigma = 5$. Find $\Pr[X < 106]$.

A: 1.2	B: 1.0	C: .1151	D: .3849	E: .8849

53. If X is a normal random variable with mean $\mu = 1000$ and standard deviation $\sigma = 10$, what is $\Pr[990 < X < 1020]$?

A: .03	B: .1359	C: .8185	D: .0073	E: .9544

54. The number of Wriggly Jiggly candies in a Super Jumbo size carton is approximately normal, with mean $\mu = 60$ and standard deviation $\sigma = 5$. What is the probability that a particular carton contains more than 65 Wriggly Jiggly candies?

A: .1841	B: .1587	C: .2119	D: .1357	E: .0968

55. A fair coin is tossed 100 times. Let X be the number of times heads comes up. Find $\Pr[X = 50]$.

A: .5	B: .0796	C: 0	D: .5398	E: .0398

Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

Student Number

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 028b Final Examination

Wednesday, April 17, 2002 Code 111 9:00 a.m. - 12:00 noon

INSTRUCTIONS

1. Check that your booklet is complete. There are 50 questions which start on Page 1 and continue to Page 7. Questions are printed on both sides of the page. The table for the Normal Distribution is provided (at the end of the exam on Page 8) and may be used when appropriate.

2. Do not unstaple the booklet. The two blank pages at the back of the booklet may be torn off and used for rough work. As well, the probability table on Page 8 may be removed.

3. Fill out the top of this page, and fill out the top of the scantron (answer) sheet. Print your name and sign the scantron sheet. Print and code in your student number. (Leave section and exam code blank.)

4. Code your answers to all questions (1-50) on the scantron sheet using an HB pencil.

5. The 50 questions are each worth 1 mark, for a total of 50 marks.

6. This question booklet, your scantron sheet and all scrap paper must be handed in at the end of the exam.

7. CALCULATORS MAY BE USED.