## PART A
## (20 marks)

Circle your answer in each question below **and** mark it on the (scantron) answer sheet. Code as you go. Extra time will NOT be given for coding answers at the end of the exam. Be advised that ONLY THE SCANTRON CARD WILL BE MARKED IN THIS SECTION, but only this question paper will be returned to you.

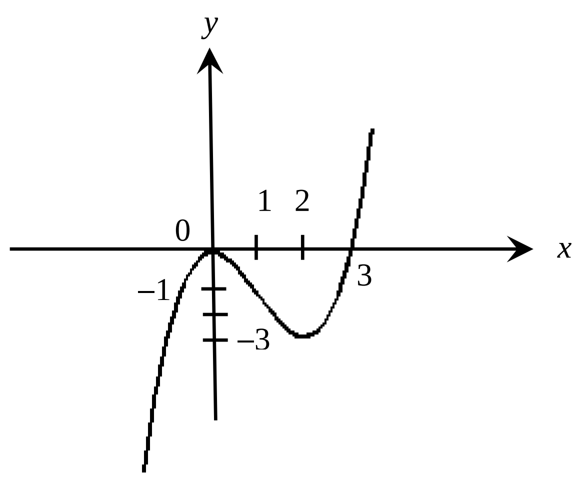
A1. Let $y$ be defined in terms of $x$ by the equation $x^2 - y^2 = 25$. Find $\frac{dy}{dx}$ by implicit differentiation.

| | | | | |
|---|---|---|---|---|
| A: $-y/x$ | B: $2x/y$ | C: $x/y$ | D: $(25-2x)/2y$ | E: $x/2y$ |

A2. Let $y$ be defined in terms of $x$ by the equation $xy^2 = 4$. Find the slope of the tangent line to the graph of this curve at the point $(1, -2)$.

| | | | | |
|---|---|---|---|---|
| A: 1 | B: 2 | C: 3 | D: 0 | E: −1 |

Use the graph of the function $f$ given below to answer questions A3 to A6.




A3. List all the $x$ values that are critical numbers of the function $f$.

| | | | | |
|---|---|---|---|---|
| A: 0, 1, 2 | B: 1 | C: 0 | D: 0, 2 | E: 2, 3 |

A4. Find the interval on which the function $f$ is decreasing.

| | | | | |
|---|---|---|---|---|
| A: $(0, 2)$ | B: $(1, 3)$ | C: $(0, 3)$ | D: $(-\infty, 1)$ | E: $(-\infty, 2)$ |

A5. Find the interval on which the function $f$ is concave downward.

| | | | | |
|---|---|---|---|---|
| A: $(0, 2)$ | B: $(0, 1)$ | C: $(0, 3)$ | D: $(-\infty, 1)$ | E: $(-\infty, 2)$ |

A6. List all the inflection points of the function $f$.

| | | | | |
|---|---|---|---|---|
| A: $(0, 0)$ | B: $(-1, 0)$ | C: $(0, 0), (1, -1)$ | D: $(1, -1)$ | E: $(2, -3)$ |

Let the function $f(x) = x^3 + 3x^2 = x^2(x+3)$ for all $x$ with first derivative $f'(x) = 3x(x+2)$ and second derivative $f''(x) = 6(x+1)$.

Use this information to answer questions A7 – A11.

A7. Find the interval on which the function $f$ is decreasing.

| | | | | |
|---|---|---|---|---|
| A: $(-2, 0)$ | B: $(-\infty, -1)$ | C: $(-\infty, 0)$ | D: $(-1, \infty)$ | E: $(-2, \infty)$ |

A8. Which of the following $x$ values is one of the critical numbers of the function $f$?

| | | | | |
|---|---|---|---|---|
| A: 2 | B: 1 | C: 0 | D: $-1$ | E: $-3$ |

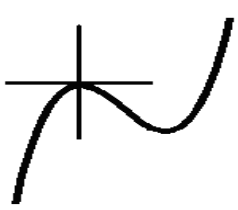
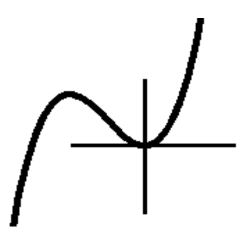
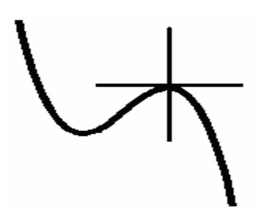
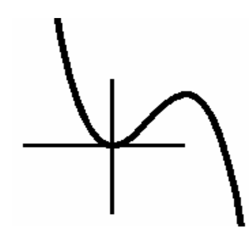
A9. Which of the following is the relative maximum value of the function $f$?

| | | | | |
|---|---|---|---|---|
| A: 0 | B: 1 | C: 2 | D: 4 | E: 6 |

A10. Find the interval on which the function $f$ is concave upward.

| | | | | |
|---|---|---|---|---|
| A: $(-2, 0)$ | B: $(-\infty, -1)$ | C: $(-\infty, \infty)$ | D: $(-2, \infty)$ | E: $(-1, \infty)$ |

A11. Which of the following illustrations is a sketch of the graph of the function $f$?
The axes indicate the location of the origin.

| | | | |
|---|---|---|---|
| A:  | B:  | C:  | D:  |

A12. Let $f$ be a function with domain $(-\infty, \infty)$ and $f(-2) = 60$, $f(4) = 48$, $f(10) = 428$,
$f'(-2) = f'(4) = 0$, $f'(10) = 216$, $f''(-2) = -18$, $f''(4) = 18$, $f''(10) = 54$.
From this information find a relative maximum value of the function $f$.

| | | | |
|---|---|---|---|
| A: 48 | B: 60 | C: 428 | D: cannot be determined |

Let $f(x)=\dfrac{2x^2+1}{x^2+1}$ with first derivative, $f'(x)=\dfrac{2x}{(x^2+1)^2}$ and second derivative, $f''(x)=\dfrac{2(1-3x^2)}{(x^2+1)^3}$.
Use this information to answer questions A13 – A20.

A13: Find the domain of the function *f*.

| | | | | |
|---|---|---|---|---|
| A: $(-\infty,\infty)$ | B: $(-\infty,-1)\cup(-1,\infty)$ | C: $(-\infty,0)\cup(0,\infty)$ | D: $(-\infty,-1)$ | E: $(-1,\infty)$ |

A14. Which of the following is the equation of the horizontal asymptote of the graph of the function *f* ?

| | | | | |
|---|---|---|---|---|
| A: $x=2$ | B: $y=2$ | C: $y=-1$ | D: $x=1$ | E: none exist |

A15. Which of the following is the equation of the vertical asymptote of the graph of the function *f* ?

| | | | | |
|---|---|---|---|---|
| A: $x=2$ | B: $y=2$ | C: $y=-1$ | D: $x=1$ | E: none exist |

A16. List all the $x$ values that are critical numbers of *f* .

| | | | | |
|---|---|---|---|---|
| A: $\frac{-1}{\sqrt{3}},0,\frac{1}{\sqrt{3}}$ | B: $\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}$ | C: 1 | D: 0 | E: none exist |

A17. Find the interval on which the function *f* is increasing.

| | | | | |
|---|---|---|---|---|
| A: $(-\infty,0)$ | B: $\left(\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ | C: $(0,\infty)$ | D: $(-\infty,\infty)$ | E: none exist |

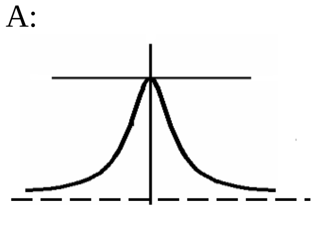
A18. Find the point at which the function *f* has a relative extremum.

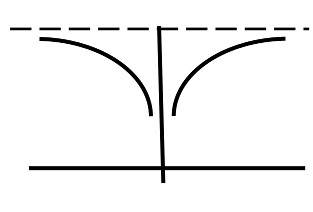
| | | | | |
|---|---|---|---|---|
| A: $\left(\frac{1}{\sqrt{3}},\frac{5}{4}\right)$ | B: $(0, 1)$ | C: $(0, 0)$ | D: $(0, 2)$ | E: none exist |

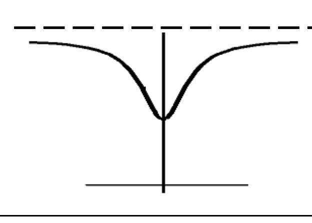
A19. On what interval is the function concave upward?

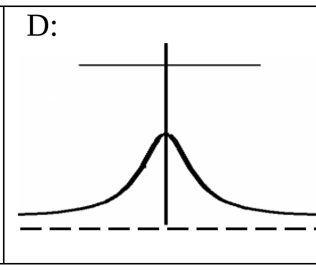
| | | | | |
|---|---|---|---|---|
| A: $(-\infty,-\frac{1}{\sqrt{3}})$ | B: $\left(\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ | C: $\left(\frac{1}{\sqrt{3}},\infty\right)$ | D: $(0,\infty)$ | E: none exist |

A20. Which of the following illustrations is a sketch of the graph of the function *f* ?
The axes indicate the location of the origin.

A:


B:


C:


D:


## PART B
## (30 marks)
## Show all your work for each question.

**MARKS**

8 B1. Let $y^2 + y = x$.

(a) Use implicit differentiation to find the first derivative $\frac{dy}{dx}$ in terms of $x$ and/or $y$.

(b) Find the second derivative $\frac{d^2y}{dx^2}$ in terms of $x$ and/or $y$. DO NOT SIMPLIFY.

**MARKS**

6 B2. Let $f(x) = x - 3x^{1/3}$.

(a) Find the values of $x$ where the first derivative, $f'(x)$, is either zero or undefined.

(b) Find the interval(s) where the function $f(x)$ is concave upward.

**MARKS**

9 B3. Let the function $f(x) = 4 - 3x^2 - x^3 = (2+x)^2(1-x)$ with $f'(x) = -3x(2+x)$ and $f''(x) = -6(1+x)$.

(a) The function $f$ has only one relative minimum value. Find it.

(b) Find the absolute maximum value and the absolute minimum values of the function $f$ on the interval $[-1, 2]$.

**MARKS**

7 B4. A ladder of length 5 metres standing on level ground is leaning against a vertical wall. The base of the ladder is pushed towards the wall. At the instant of time when the base of the ladder is 3 metres from the wall the base of the ladder is moving at 1 metre per second towards the wall. How fast is the top of the ladder rising up the wall at that instant of time?

Your solution of this problem should include a diagram, variables that are clearly defined and an equation relating the variables.

END