Mathematics 012A Test 2 November 9, 2007
Solutions

CODE 111

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
C	A	D	A	D	D	A	C	D	E	B	B	A	B	E	D	C	B	B	C

CODE 333

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	C	B	C	E	B	B	E	D	C	C	E	E	B	C	A	A	C	D

Long Answer for CODE 111

B1.

(a) $$2yy' + y' = 1 \ \rightarrow\ (2y+1)y' = 1 \ \rightarrow\ y' = \frac{1}{2y+1}$$

(b) $$y'' = \frac{-1}{(2y+1)^2}(2y') = \frac{-2\left(\frac{1}{2y+1}\right)}{(2y+1)^2} = \frac{-2}{(2y+1)^3}$$

B2.

(a) $$f'(x) = 1 - 3\left(\frac{1}{3}\right)x^{-2/3} = 1 - x^{-2/3} = 1 - \left(\frac{1}{x^2}\right)^{1/3}$$

$f'(x) = 0$ when $x = 1$ or $x = -1$

$f'(x)$ is undefined when $x = 0$

(b) $f''(x) = \frac{2}{3}x^{-5/3} \ \rightarrow\ f''(x)$ is undefined at $x = 0$ and equal to zero nowhere

∩		∪
$f'' < 0$	**0**	$f'' > 0$

f is concave upward on $(0, \infty)$

B3.

(a) f has critical numbers 0 and -2.

$f''(0) = -6$ and $f''(-2) = 6$

So f has a relative minimum at $x = -2$ by the second derivative test.

That relative minimum value is $f(-2) = 0$.

(b) f has one critical number in $[-1, 2]$, namely 0.

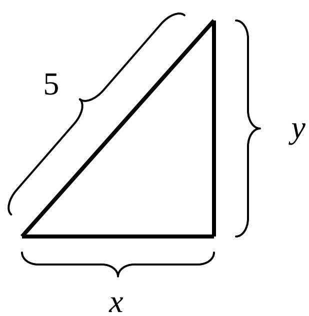
We consider $f(-1) = 2$ and $f(2) = -16$ – at the endpoints of $[-1, 2]$

and $f(0) = 4$ – at the critical number 0.

It is clear that the absolute maximum value of f on $[-1, 2]$ is 4 at $x = 0$ and the absolute minimum value of f on $[-1, 2]$ is -16 at $x = 2$.

B4.

1. Diagram with variables x and y clearly defined.



x is the distance of the base of the ladder from the wall

y is the height of the ladder against the wall

2. $x^2 + y^2 = 25$ where $x = x(t)$ and $y = y(t)$

3. $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0 \quad \rightarrow \quad \dfrac{dy}{dt} = -\dfrac{x}{y}\dfrac{dx}{dt}$

4. When $x = 3$, $y = \sqrt{5^2 - 3^2} = 4$ and

5. $\dfrac{dx}{dt} = -1$.

6. So $\dfrac{dy}{dt} = -\dfrac{x}{y}\dfrac{dx}{dt} = -\dfrac{3}{4}(-1) = \dfrac{3}{4}$.

7. That is, the ladder is rising up the wall at 3/4 m/s.