Mathematics 030 Midyear Examination Monday, December 12, 2005

1. Find the value of $5^{\log_5 7}$.

A: 5	B: 7	C: 35	D: 7^5	E: 5^7

2. Simplify $\dfrac{\log_2 36 - \log_2 4}{\log_2 27}$.

A: $\dfrac{32}{27}$	B: $\dfrac{2}{3}$	C: $\dfrac{1}{3}$	D: $\log_2\left(\dfrac{1}{3}\right)$	E: $\log_2\left(\dfrac{32}{27}\right)$

3. If $f(x) = x \ln x$ then $f'(e) =$

A: 0	B: 1	C: 2	D: $\dfrac{1}{e}$	E: e

4. If $f(x) = 3^{2x}$ find $f'(x)$.

A: $(2x)3^{2x-1}$	B: 3^{2x}	C: $2(3^{2x})$	D: $2(\ln 3)3^{2x}$	E: $(\ln 3)3^{2x}$

5. If $f(x) = x^{\ln x}$ find $f'(x)$.

A: $(\ln x)\left(x^{\ln x - 1}\right)$	B: $\left(x^{\ln x}\right)(\ln x)$	C: $\left(x^{\ln x}\right)\left(\dfrac{1}{x}\right)$	D: $\left(2x^{\ln x}\right)\left(\dfrac{\ln x}{x}\right)$	E: $\dfrac{1}{x}$

6. Determine the x-coordinate of the point on the curve $y = 8\ln(x+1)$ where the tangent line is parallel to the line $y = 2x + 5$.

A: -1	B: 0	C: 1	D: 2	E: 3

7. Given that $f(x) = xe^x$, $f'(x) = (x+1)e^x$ and $f''(x) = (x+2)e^x$, find the interval on which $f(x)$ is increasing.

A: $(-\infty, -1)$ only	B: $(-1, \infty)$ only	C: $(-\infty, -2)$ only
D: $(-2, \infty)$ only	E: $(-2, -1)$ only	

8. If $f'(x) = 3x^2 + 3$ and $f(0) = 4$ then $f(1) =$

A: 8	B: 7	C: 6	D: 5	E: 4

9. If $\dfrac{8x - 11}{(x-2)(x-1)} = \dfrac{A}{x-2} + \dfrac{B}{x-1}$, find the value of A.

A: 1	B: 2	C: 3	D: 4	E: 5

10. Find $\displaystyle\int \frac{1}{(x+1)^2}\,dx$.

A: $\ln(x+1)^2 + C$	B: $\dfrac{3}{(x+1)^3} + C$	C: $\dfrac{(x+1)^3}{3} + C$	D: $-\dfrac{1}{x+1} + C$	E: $-\ln\lvert x+1\rvert + C$

11. Find $\displaystyle\int \frac{4x}{x^2+1}\,dx$.

A: $4\ln(x^2+1) + C$	B: $2\ln(x^2+1) + C$	C: $4\ln\lvert x\rvert + 2x^2 + C$
D: $\dfrac{2x^2}{x^3+x} + C$	E: $\dfrac{4x^2}{(x^2+1)^2} + C$	

12. Find $\displaystyle\int \frac{4(1+\ln x)^3}{x}\,dx$.

A: $16(1+\ln x)^4+C$	B: $4(1+\ln x)^4+C$	C: $(1+\ln x)^4+C$
D: $12(1+\ln x)^2+C$	E: none of A,B,C,D	

13. Evaluate $\int_0^1 (1-3x)^5 dx$.

A: $\frac{1}{6}$	B: $\frac{63}{18}$	C: $-\frac{63}{18}$	D: $\frac{63}{6}$	E: $-\frac{63}{6}$

14. Evaluate $\int_0^1 xe^{(x^2)} dx$.

A: $2e-2$	B: $2e$	C: $\frac{e}{2}$	D: $\frac{e}{2}-1$	E: $\frac{e}{2}-\frac{1}{2}$

15. Evaluate $\int_0^3 xe^x dx$.

A: e^3+1	B: e^3-1	C: $2e^3$	D: $2e^3+1$	E: $2e^3-1$

16. Find the area of the region bounded by the curves $y=e^{2x}$ and $y=0$ between $x=\frac{1}{2}$ and $x=1$.

A: $e-e^{\frac{1}{2}}$	B: $\frac{e}{2}-\frac{e^{\frac{1}{2}}}{2}$	C: $e-e^{\frac{1}{4}}$	D: $\frac{e^2}{2}-\frac{e}{2}$	E: e^2-e

17. Find the area of the region bounded by the curves $y=3x^2$ and $y=x^3$.

A: 54	B: $\frac{54}{4}$	C: $\frac{27}{4}$	D: $\frac{3}{4}$	E: 0

18. Find the volume of the solid generated when the region bounded by the curves $y=x^3$, $y=0$ and $x=1$ is rotated about the x-axis.

A: $\frac{\pi}{7}$	B: $\frac{\pi}{6}$	C: $\frac{\pi}{5}$	D: $\frac{\pi}{4}$	E: $\frac{\pi}{3}$

Use the following diagram in questions 19, 20 and 21.

19. Which of the stated integrals gives the area of the shaded region?

A: $\int_0^1 3x^2 dx$	B: $\int_0^9 (3-3x^2)dx$	C: $\int_0^3 (3-3x^2)dx$
D: $\int_0^3 \sqrt{\frac{y}{3}}\,dy$	E: $\int_0^3 \frac{\sqrt{y}}{3}\,dy$	

20. Which of the stated integrals gives the volume of the solid generated when the shaded region is rotated about the x-axis?

A: $\pi\int_0^3 \frac{y}{3}\,dy$	B: $\pi\int_0^3 \sqrt{\frac{y}{3}}\,dy$	C: $\pi\int_0^1 9x^4 dx$
D: $\pi\int_0^1 (3-3x^2)^2 dx$	E: $\pi\int_0^1 (9-9x^4)dx$	

21. Which of the stated integrals gives the volume of the solid generated when the shaded region is rotated about the y-axis?

A: $\pi\int_0^3 \frac{y}{3}\,dy$	B: $\pi\int_0^3 \sqrt{\frac{y}{3}}\,dy$	C: $\pi\int_0^1 9x^4 dx$
D: $\pi\int_0^1 (3-3x^2)^2 dx$	E: $\pi\int_0^1 (9-9x^4)dx$	

22. Evaluate the improper integral $\int_2^\infty e^{-2x} dx$.

A: $-\frac{1}{2e^4}$	B: $\frac{1}{2e^4}$	C: $-\frac{1}{e^4}$	D: $\frac{1}{e^4}$	E: diverges

23. If $f(x) = \sec 5x$ find $f'(x)$.

A: $5\sec x\tan x$	B: $5\sec 5x\tan 5x$	C: $25\sec 5x\tan 5x$	D: $-5\sec x\tan x$	E: $\sec 5x$

24. If $f(x) = x\cos x$ then $f'(\pi) =$

A: π	B: $-\pi$	C: 1	D: -1	E: 0

25. If $f(x) = \sin(\pi x)$ then $f''\left(\frac{1}{2}\right) =$

A: -1	B: 0	C: 1	D: π^2	E: $-\pi^2$

26. If $f(x) = \cos^2 x$ find the slope of the tangent line to the graph of $y = f(x)$ at $x = \frac{\pi}{6}$.

A: $-\frac{1}{4}$	B: $\frac{1}{4}$	C: $\frac{3}{4}$	D: $\frac{\sqrt{3}}{2}$	E: $-\frac{\sqrt{3}}{2}$

27. $\int \frac{d}{dx}(\sin x)dx =$

A: $\sin x + C$	B: $-\sin x + C$	C: $\cos x + C$	D: $-\cos x + C$	E: $\frac{1}{2}(\sin x)^2 + C$

28. $\int x\sin x\,dx =$

A: $-x\cos x + \int \cos x\,dx$	B: $x\cos x - \int \cos x\,dx$	C: $-x\cos x - \int \cos x\,dx$
D: $x\sin x - \int \sin x\,dx$	E: $x\sin x + \int \sin x\,dx$	

29. Evaluate $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x\,dx$.

A: 1	B: $\frac{1}{2}$	C: $\frac{1}{3}$	D: $\frac{1}{4}$	E: $\frac{1}{\sqrt{2}}$

30. Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}\,dx$.

A: $\ln\left(\frac{2}{3}\right)$	B: $\ln\left(\frac{3}{2}\right)$	C: -1	D: 0	E: 1

31. If $f(x,y,z) = ze^{xy}\ln y$, find $f(-1,\sqrt{e},5)$.

A: $\frac{5}{\sqrt{e}}$	B: $\frac{5\sqrt{e}}{e^{\sqrt{e}}}$	C: $\frac{5}{2e^{\sqrt{e}}}$	D: $-\frac{5e^{\sqrt{e}}}{2}$	E: $-\frac{\ln 5}{e^{\sqrt{e}}}$

32. If $f(x,y,z) = ze^{xy}\ln y$, find $f_x(x,y,z)$.

A: ye^{xy}	B: $ze^{x}\ln y$	C: $ze^{xy}\ln y$	D: $yze^{xy}\ln y$	E: $\frac{ze^{xy}}{y}$

33. Find $f_y(1,2)$ where $f(x,y) = x^2y^3 - 3x^3y$.

A: -12	B: -2	C: 2	D: 9	E: 15

34. Find $f_{xx}(x,y)$ where $f(x,y) = x^3y^2 - xy^3$.

A: $6x$	B: $6xy^2$	C: $6xy^2 - y^3$	D: $6xy^2 - 3y^2$	E: $6x^2y - 3y^2$

35. If $f(x,y) = x^2\sqrt{y} + \ln(xy)$, find $f_{xy}(x,y)$.

A: $2x$	B: $\frac{x}{\sqrt{y}}$	C: $2x\sqrt{y} + \frac{1}{x}$	D: $\frac{x}{\sqrt{y}} - \frac{1}{xy^2}$	E: $\frac{x^2}{2\sqrt{y}} + \frac{1}{y}$

36. Find all critical points of the function $f(x,y) = x^3 - 3xy^2 + 3y^2$.

A: $(0,0)$	B: $(1,-1),(1,1)$
C: $(0,0),(1,-1),(1,1)$	D: $(1,-1),(1,1),(-1,1),(-1,-1)$
E: $(0,0),(1,-1),(1,1),(-1,1),(-1,-1)$	

Use the following information for questions 37, 38 and 39.

$$\begin{aligned} f(x,y) &= x^3 - y^3 - 27x + 3y \\ f_x(x,y) &= 3x^2 - 27 \\ f_y(x,y) &= -3y^2 + 3 \\ f_{xx}(x,y) &= 6x \\ f_{xy}(x,y) &= 0 \\ f_{yy}(x,y) &= -6y \end{aligned}$$

37. Which one of the following is true for the point $(3,1)$?

A: $(3,1)$ is not a critical point of $f(x,y)$.	B: There is a local maximum at $(3,1)$.
C: There is a local minimum at $(3,1)$.	D: There is a saddle point at $(3,1)$.

38. Which one of the following is true for the point $(-3,1)$?

A: $(-3,1)$ is not a critical point of $f(x,y)$.	B: There is a local maximum at $(-3,1)$.
C: There is a local minimum at $(-3,1)$.	D: There is a saddle point at $(-3,1)$.

39. Which one of the following is true for the point $(1,-3)$?

A: $(1,-3)$ is not a critical point of $f(x,y)$.	B: There is a local maximum at $(1,-3)$.
C: There is a local minimum at $(1,-3)$.	D: There is a saddle point at $(1,-3)$.

40. Consider the problem of finding the minimum of the function $f(x,y) = x + 3y$ subject to the constraint that $x^2 + y^2 = 10$. At what point (x,y) does the minimum value occur?

A: $(1,3)$	B: $(-1,-3)$	C: $(-1,3)$	D: $(1,-3)$	E: $(0,0)$

41. A rectangular box with top, bottom and sides, which has 8 internal compartments (3 dividers widthwise and 1 divider lengthwise) is to be constructed. If there are 100 square centimetres of material available and the volume of the box is to be maximized, then the problem is to maximize $f(x,y,z) = xyz$ subject to the constraint that $2xy + 5xz + 3yz = 100$. To solve this problem using Lagrange's method, what system of equations must be solved?

A: $xyz + \lambda(2xy + 5xz + 3yz - 100) = 0$	B: $\begin{aligned} xyz + \lambda(2y+5z) &= 0 \\ xyz + \lambda(2x+3z) &= 0 \\ xyz + \lambda(5x+3y) &= 0 \\ 2xy + 5xz + 3yz &= 100 \end{aligned}$
C: $\begin{aligned} yz + 2y\lambda &= 0 \\ xz + 2x\lambda &= 0 \\ xy + 5x\lambda &= 0 \\ 2xy + 5xz + 3yz &= 100 \end{aligned}$	D: $\begin{aligned} yz + \lambda(2y+5z) &= 0 \\ xz + \lambda(2x+3z) &= 0 \\ xy + \lambda(5x+3y) &= 0 \\ 2xy + 5xz + 3yz &= 100 \end{aligned}$
E: $\begin{aligned} yz + \lambda(2xy + 5xz + 3yz - 100) &= 0 \\ xz + \lambda(2xy + 5xz + 3yz - 100) &= 0 \\ xy + \lambda(2xy + 5xz + 3yz - 100) &= 0 \end{aligned}$	

42. Find the general solution to the first order differential equation $\dfrac{dy}{dx} = \dfrac{\cos x}{3y^2}$.

A: $y = (\sin x)^{\frac{1}{3}} + C$	B: $y = (\sin x + C)^{\frac{1}{3}}$	C: $y = \frac{1}{3}(\sin x)^{\frac{1}{3}} + C$
D: $y = (-\sin x + C)^{\frac{1}{3}}$	E: $y = -\frac{1}{3}(\sin x)^{\frac{1}{3}} + C$	

43. Find y, where $\frac{dy}{dx} = e^x y^2$ and it is known that $y(\ln 2) = 1$.

A: $y = \frac{1}{e^x - 1}$	B: $y = -\frac{1}{e^x - 3}$	C: $y = -\frac{1}{e^x} + \frac{3}{2}$	D: $y = -\frac{1}{e^x} + \frac{2}{3}$	E: $y = \frac{1}{\ln 2}$

44. Which of the following is an <u>integrating factor</u> that can be used to solve $\frac{dy}{dx} + x^2 y = e^x$?

A: e^x	B: $e^{(x^2)}$	C: $e^{(e^x)}$	D: $\frac{x^3}{3}$	E: $e^{(x^3/3)}$

45. Find $y(1)$ where it is known that $\frac{dy}{dx} - y = xe^{2x}$ and $y(0) = 1$, using the integrating factor $I(x) = e^{-x}$.

A: $2e$	B: 2	C: $\frac{e^2}{2} + e$	D: $\frac{e^3 + 1}{4}$	E: $\frac{2e^2 + 1}{9}$

46. Find $y(3)$ if $\frac{dy}{dt} = 5y$ and $y(0) = 2$.

A: $6e^{15}$	B: $15e^2$	C: $2e^{15}$	D: $5e^6$	E: $6e^5$

47. If $\frac{dy}{dt} = ky$ and it is known that $y(1) = 2y(0)$, what is the value of k?

A: $\frac{1}{2}$	B: $\ln\left(\frac{1}{2}\right)$	C: 2	D: $\ln 2$	E: cannot be determined

48. The population $y(t)$ of a certain city increases exponentially over time (t), according to the differential equation $\frac{dy}{dt} = ky$. In January 2000, the population was 3 million and in January 2005 the population was 4 million. What will the population of this city be in January 2010?

A: $4\frac{1}{3}$ million	B: 5 million	C: $5\frac{1}{3}$ million
D: 6 million	E: cannot be determined	

49. The slope of a certain curve at the point (x, y) is given by $\frac{3x^2 - 2x}{y}$. This curve passes through the point $(1, 2)$. Find an equation for the curve.

A: $\ln\lvert y\rvert = x^3 - x^2 + 2$	B: $y^2 = x^3 - x^2 + 2$	C: $y^2 = x^3 - x^2$
D: $y^2 = 2x^3 - 2x^2 + 2$	E: $y^2 = 2x^3 - 2x^2 + 4$	

50. Blood enters and leaves Jack's liver at the rate of 3.5 cubic centimetres per second. The capacity of Jack's liver is 350 cubic centimetres of blood. Determine a mathematical model for the amount $y(t)$ of a drug (in grams) in Jack's previously drug-free liver, t seconds after blood carrying the drug, in a concentration of 0.1 grams per cubic centimetre, first enters the liver.

A: $\begin{aligned}\frac{dy}{dt} &= 0.35 - \frac{y}{100} \\ y(0) &= 0\end{aligned}$	B: $\begin{aligned}\frac{dy}{dt} &= 3.5 - \frac{y}{350} \\ y(0) &= 0\end{aligned}$	C: $\begin{aligned}\frac{dy}{dt} &= 35 - \frac{y}{35} \\ y(0) &= 0\end{aligned}$
D: $\begin{aligned}\frac{dy}{dt} &= 0.35 - 3.5y \\ y(0) &= 0\end{aligned}$	E: $\begin{aligned}y &= 350e^{0.35t} \\ y(0) &= 0\end{aligned}$	