Mathematics 030 Midyear Examination Monday, December 13, 2004

1. Express $\dfrac{\log_2 8 - \log_3 9}{\log_4 2}$ as simply as possible.

A: $\dfrac{1}{2}$	B: $\dfrac{3}{4}$	C: 2	D: 3	E: $\log_2\left(\dfrac{2}{9}\right)$

2. If $2^{3x-1} = 16$, what is the value of x?

A: 5	B: 1	C: 4	D: $\dfrac{4}{3}$	E: $\dfrac{5}{3}$

3. If $f(x) = 3e^x - 2\ln x + \dfrac{1}{4x}$, what is $f'(x)$?

A: $3e^x - \dfrac{2}{x} - \dfrac{1}{4x^2}$	B: $3e^x - \dfrac{2}{x} + \dfrac{1}{4}$	C: $e^{3x} - \dfrac{2}{x} - 2\ln x + \dfrac{1}{2x^2}$
D: $3e^x - \dfrac{1}{x^2} + \ln(4x)$	E: $3e^x - \dfrac{2}{x} - \dfrac{1}{16x^2}$	

4. If $f(x) = \dfrac{e^x}{2} - x^2$, find $f'(\ln 4)$.

A: $2 - \ln 8$	B: $2 - \ln 16$	C: $2 - (\ln 4)^2$	D: $-3\ln 2$	E: $\dfrac{3\ln 4}{2}$

5. If $f(x) = [\ln(3x+e)]^{2e}$, find $f'(x)$.

A: $\left(\dfrac{1}{3x+e}\right)^{2e}$	B: $\left(\dfrac{3}{3x+e}\right)^{2}$	C: $\dfrac{6e}{3x+e}[\ln(3x+e)]^2$
D: $\dfrac{6e}{3x+e}[\ln(3x+e)]^{2e-1}$	E: $\dfrac{8e}{3x+e}[\ln(3x+e)]^{2e}$	

6. Find $f'(x)$, where $f(x) = 2^x$.

A: 2	B: 2^x	C: $2^x \ln x$	D: $2^x \ln 2$	E: $\dfrac{1}{2^x \ln 2}$

7. Find $\dfrac{d}{dx}(\log_5 x^2)$.

A: $\dfrac{2}{x}$	B: $\dfrac{2}{x\ln 5}$	C: $x^2 \ln 5$	D: $\dfrac{1}{x^2 \ln 5}$	E: $2x^3 \ln 5$

8. If $f'(x) = 2e^x + 2x + \dfrac{2}{x}$, what is $f(x)$?

A: $2e^x + 2 - \dfrac{2}{x^2}$	B: $e^{2x} + x^2 + 2 + C$	C: $2e^x + x^2 + 2\ln\lvert x\rvert + C$
D: $2e^x + x^2 + \ln\left\lvert\dfrac{x}{2}\right\rvert + C$	E: $2(e^x + x^2 + \ln\lvert x\rvert) + C$	

9. Find $\displaystyle\int x\sqrt{x^2+1}\,dx$.

A: $\sqrt{2x}+C$	B: $\dfrac{(x^2+1)^{3/2}}{3}+C$	C: $\dfrac{x^2(x^2+1)^{3/2}}{3}+C$
D: $\dfrac{1}{2\sqrt{x^2+1}}+C$	E: $\dfrac{x^2}{4\sqrt{x^2+1}}+C$	

10. If $f(x)=\dfrac{3x^2-1}{x^3-x+5}$, what is $\displaystyle\int f(x)dx$?

A: $\ln\lvert x^3-x+5\rvert+C$	B: $(3x^2-1)\ln\lvert x^3-x+5\rvert+C$	C: $(x^3-x)\ln\lvert x^3-x+5\rvert+C$
D: $\dfrac{3x^2-1}{x^3-x+5}+C$	E: $\dfrac{x^3-x}{x^3-x+5}+C$	

11. What is $\displaystyle\int xe^{2x}dx$?

A: $3xe^{2x}+C$	B: $x^2e^{2x}+C$	C: $e^{2x}(x-1)+C$	D: $\dfrac{x^2e^{2x}}{4}+C$	E: $\dfrac{e^{2x}}{2}\left(x-\dfrac{1}{2}\right)+C$

12. If $\dfrac{3x-1}{x^2-3x+2}=\dfrac{A}{x-2}+\dfrac{B}{x-1}$, find the value of A.

A: 5	B: 2	C: -5	D: -2	E: 1

13. If $f(x)=\ln(x^2+1)$, evaluate $\displaystyle\int_{-1}^{2} f'(x)dx$.

A: $\ln 10+\ln 2$	B: $\ln\left(\dfrac{5}{2}\right)$	C: $\ln 3$	D: $\ln 7$	E: $\dfrac{9}{5}$

14. Evaluate $\displaystyle\int_{0}^{\ln 2} e^{3x+\ln 2}dx$.

A: $\dfrac{10}{3}$	B: $\dfrac{14}{3}$	C: $\dfrac{16}{3}$	D: 6	E: 14

15. Evaluate $\displaystyle\int_{1}^{2} 3x^2\ln x\,dx$.

A: $8\ln 2$	B: $8\ln 2-4$	C: $8\ln 2-\dfrac{7}{3}$	D: $\ln 16-7$	E: $12\ln 2+3$

16. Find the area of the region bounded by $y=x^2$ and the x-axis between $x=1$ and $x=3$.

A: $\dfrac{28}{3}$	B: 9	C: $\dfrac{26}{3}$	D: 8	E: 10

17. Find the area of the region bounded by the curves $y=x^4$ and $y=x$.

A: $\dfrac{3}{10}$	B: $\dfrac{1}{2}$	C: $\dfrac{1}{5}$	D: $\dfrac{2}{5}$	E: $\dfrac{7}{10}$

18. Find the volume generated when the region bounded by the curves $y=x^2, y=0$, and $x=1$ is rotated about the x-axis.

A: $\dfrac{1}{3}\pi$	B: π	C: $\dfrac{4}{5}\pi$	D: $\dfrac{2}{3}\pi$	E: $\dfrac{1}{5}\pi$

Use the following diagram in questions 19 and 20.

19. Determine an integral that represents the volume generated when the shaded region is revolved about the x-axis.

A: $\pi \int_0^1 (e - e^x)dx$	B: $\pi \int_0^1 (e^2 - e^{2x})dx$	C: $\pi \int_0^1 (e^{2x} - e^2)dx$
D: $\pi \int_0^1 (e^x - e)dx$	E: $\pi \int_0^1 (e - e^{2x})dx$	

20. Determine an integral that represents the volume generated when the shaded region is revolved about the y-axis.

A: $\pi \int_0^1 e^{2y}dy$	B: $\pi \int_1^e e^{2y}dy$	C: $\pi \int_0^1 (\ln y)^2 dy$
D: $\pi \int_1^e (\ln y)^2 dy$	E: $\pi \int_1^e \ln y\, dy$	

21. Evaluate the improper integral $\int_0^\infty e^{-5x}dx$.

A: 5	B: 0	C: $\frac{1}{5}$	D: 1	E: diverges

22. Evaluate the improper integral $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$.

A: $\frac{1}{2}$	B: 2	C: $-\frac{1}{2}$	D: -2	E: diverges

23. If $f(x) = 1 + 2\cos x$, find $f'\left(\frac{\pi}{6}\right)$.

A: $-\sqrt{3}$	B: $\sqrt{3}$	C: $-\sqrt{2}$	D: 1	E: -1

24. If $f(x) = 3\sec^2 x$, what is $f'(x)$?

A: $\sec^3 x$	B: $3\tan^2 x$	C: $6\sec x \tan x$	D: $6\sec^2 x \tan x$	E: $6\sec(x^2)\tan x$

25. Find $f'(x)$, where $f(x) = \ln(\sin(x^2))$.

A: $\sec 2x$	B: $\csc(x^2)$	C: $\cot(x^2)$	D: $2x\tan(x^2)$	E: $2x\cot(x^2)$

26. Find the slope of the tangent line to the curve $y = \tan x$ at the point $\left(\frac{\pi}{3}, \sqrt{3}\right)$.

A: $\frac{1}{4}$	B: 4	C: $\frac{4}{3}$	D: $\sqrt{3}$	E: $2\sqrt{3}$

27. If $g'(x) = \sec(x+\pi)\tan(x+\pi)$ and $g(-\pi) = 2$, find $g(x)$.

A: $\sec(x+\pi)+1$	B: $\sec(x+\pi)+2$	C: $\tan(x+\pi)+2$
D: $\sec(x+\pi+1)$	E: undefined	

28. Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin x\, dx$.

A: $\frac{1}{2}$	B: $\frac{1}{\sqrt{2}}$	C: $-\frac{1}{\sqrt{2}}$	D: $1-\frac{1}{\sqrt{2}}$	E: $\frac{1}{\sqrt{2}}-1$

29. Find $\displaystyle\int \cos(\sin x)\cos x\, dx$.

A: $\sin(\sin x)+C$	B: $-\sin(\sin x)+C$	C: $\sin(\cos x)+C$
D: $-\cos(\cos x)+C$	E: $-\sin(\cos x)\sin x+C$	

30. Evaluate $\displaystyle\int_{\frac{\pi^2}{4}}^{\pi^2} \frac{\sin\sqrt{x}}{2\sqrt{x}}\, dx$.

A: 0	B: $\frac{1}{2}$	C: 1	D: -1	E: $\cos\left(\frac{\pi^2}{4}\right)-\cos(\pi^2)$

31. Find $\dfrac{\partial w}{\partial y}$ where $w = xe^{yz}$.

A: xe^{yz}	B: xe^{z}	C: e^{z}	D: xze^{yz}	E: e^{yz}

32. Find $f_x(3,-2)$ where $f(x,y) = x^2y + 2x + 3y$.

A: -10	B: 6	C: 8	D: -8	E: -18

33. Find f_{xx} where $f(x,y) = \ln(xy+1)$.

A: $\frac{y}{xy+1}$	B: $-\frac{y^2}{(xy+1)^2}$	C: $-\frac{1}{(xy+1)^2}$	D: $\frac{1}{xy+1}$	E: $-\frac{1}{x^2}$

34. Find <u>all</u> the critical points of $f(x,y) = xy - x^2 - \frac{1}{8}y^4$.

A: $(0,0)$	B: $(0,0),(2,4),(-2,-4)$	C: $(0,0),\left(\frac{1}{2},1\right),\left(-\frac{1}{2},-1\right)$
D: $(0,0),\left(\frac{1}{2},1\right)$	E: $(0,0),(2,4)$	

Use the following information in questions 35, 36 and 37.

$$\begin{aligned} f(x,y) &= \frac{1}{3}x^3 - x + xy^2 \\ f_x &= x^2 - 1 + y^2 \\ f_y &= 2xy \\ f_{xx} &= 2x \\ f_{xy} &= 2y \\ f_{yy} &= 2x \end{aligned}$$

35. Which one of the following is true for the point $(1,0)$?

A: $(1,0)$ is not a critical point of $f(x,y)$.
B: There is a local maximum at $(1,0)$.
C: There is a local minimum at $(1,0)$.
D: There is a saddle point at $(1,0)$.

36. Which one of the following is true for the point $(0,1)$?

A: $(0,1)$ is not a critical point of $f(x,y)$.
B: There is a local maximum at $(0,1)$.
C: There is a local minimum at $(0,1)$.
D: There is a saddle point at $(0,1)$.

37. Which one of the following is true for the point $(-1,0)$?

A: $(-1,0)$ is not a critical point of $f(x,y)$.
B: There is a local maximum at $(-1,0)$.
C: There is a local minimum at $(-1,0)$.
D: There is a saddle point at $(-1,0)$.

38. Suppose we use Lagrange's method to minimize $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraint $x + 2y + 2z = 18$. What system of equations must we solve?

A: $\begin{aligned} x^2 + \lambda x &= 0 \\ y^2 + 2\lambda y &= 0 \\ z^2 + 2\lambda z &= 0 \\ x + 2y + 2z &= 18 \end{aligned}$	B: $\begin{aligned} 2x + \lambda &= 0 \\ 2y + 2\lambda &= 0 \\ 2z + 2\lambda &= 0 \\ x + 2y + 2z - 18 &= 0 \end{aligned}$
C: $x^2 + y^2 + z^2 + \lambda(x + 2y + 2z - 18) = 0$	D: $\begin{aligned} 2x + \lambda x &= 0 \\ 2y + 2\lambda y &= 0 \\ 2z + 2\lambda z &= 0 \\ x + 2y + 2z - 18 &= 0 \end{aligned}$
E: $\begin{aligned} x + \lambda &= 0 \\ y + \lambda &= 0 \\ z + \lambda &= 0 \\ x + 2y + 2z - 18 &= 0 \end{aligned}$	

39. What is the minimum value of $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraint $x + 2y + 2z = 18$?

A: 10	B: 0	C: 18	D: 36	E: 12

40. A rectangular box is to have a volume of 50 cubic meters. The cost of construction is \$2 per square meter for the lid, \$4 per square meter for the sides, and \$7 per square meter for the bottom. In order to find the dimensions of the box which minimize the cost of construction, what is the mathematical problem that must be solved? (Let x, y, z be the indicated lengths in meters shown in the picture below.)

A: Minimize $C = 9xy + 4xz + 4yz$ where $xyz = 50$.
B: Minimize $C = 2xy + 2xz + 2yz$ where $xyz = 50$.
C: Minimize $C = xyz$ where $2xy + 2xz + 2yz = 50$.
D: Minimize $C = 2xy + 4xz + 4yz$ where $xyz = 50$.
E: Minimize $C = 9xy + 8xz + 8yz$ where $xyz = 50$.

41. If $\frac{dy}{dt} = 3y$ and $y(0) = 5$, find $y(2)$.

A: $5e^6$	B: $3e^{10}$	C: $3e^2 + 5$	D: $5e^2 + 3$	E: $6e^5$

42. Find k where $\frac{dy}{dt} = ky$, $y(0) = 4$ and $y(5) = 3$.

A: $\frac{3}{4}$	B: $\ln \frac{3}{4}$	C: $\frac{1}{5} \ln \frac{3}{4}$	D: 4	E: $\frac{1}{5} \ln \frac{4}{3}$

43. The population of a certain country is 20 million on January 1, 2000 and is 21 million on January 1, 2003. Assume that the exponential growth model is appropriate. Let $y(t)$ be the population of the country in millions, t years after January 1, 2000. Construct the mathematical model for this situation.

A: $\begin{aligned} \frac{dy}{dt} &= ke^y \\ y(0) &= 20 \\ y(3) &= 21 \end{aligned}$	B: $\begin{aligned} \frac{dy}{dt} &= ky \\ y(0) &= 20 \\ y(3) &= 21 \end{aligned}$	C: $\begin{aligned} \frac{dy}{dt} &= k \ln y \\ y(0) &= 20 \\ y(3) &= 21 \end{aligned}$
D: $\begin{aligned} \frac{dy}{dt} &= y + k \\ y(0) &= 20 \\ y(3) &= 21 \end{aligned}$	E: none of A,B,C or D	

44. Solve for y if $\frac{dy}{dx} = \frac{e^x}{e^y}$.

A: $y = x + C$	B: $y = e^x + C$	C: $y = \ln(e^x + C)$	D: $y = Ce^x$	E: $y = \ln x + C$

45. Which of the following is an <u>integrating factor</u> that can be used to solve $\dfrac{dy}{dx} + \dfrac{3}{x}y = \sin x$?

A: $3\ln x$	B: $3x$	C: $e^{\frac{3}{x}}$	D: $\dfrac{3}{x}$	E: x^3

46. Find $y\left(\dfrac{\pi}{4}\right)$ where $\dfrac{dy}{dx} + (\tan x)y = \dfrac{1}{\sec x}$, $y(0) = 1$, given that an <u>integrating factor</u> for the differential equation is $\sec x$.

A: $\dfrac{\pi}{4\sqrt{2}}$	B: $\sqrt{2}$	C: $\sqrt{2}\left(\dfrac{\pi}{4}+1\right)$	D: $\dfrac{1}{\sqrt{2}}\left(\dfrac{\pi}{4}+1\right)$	E: $\dfrac{\sqrt{2}}{4}\pi$

47. The slope of a curve at the point (x, y) is equal to $\dfrac{2x}{3y^2+1}$ and the curve passes through the point $(1, 2)$. Find an equation for the curve.

A: $y^3 + y = x^2 + 9$	B: $y^3 = x^2 + 7$	C: $\ln(3y^2+1) = x^2 - 1 + \ln 13$
D: $\ln(y^3 + y) = x^2 - 1 + \ln 10$	E: none of A,B,C or D	

48. A dense object is dropped from a helicopter flying at an altitude of 10,000 feet. Assume that air resistance can be ignored and that the acceleration of gravity g is 32 feet per second per second. What is the altitude of the object 10 seconds after it is dropped?

A: 7200 feet	B: 4800 feet	C: 3400 feet	D: 5600 feet	E: 8400 feet

49. The rate at which an epidemic spreads in a city is jointly proportional to the number of people who are infected and the number of people who are not infected. Let $P(t)$ be the number of people infected, at time t days after the first reported infection, for a city of 100,000 people. Construct the mathematical model for this situation.

A: $\begin{aligned}\frac{dP}{dt} &= kP(1-P)\\ P(0) &= 1\end{aligned}$	B: $\begin{aligned}\frac{dP}{dt} &= kP(100,000-P)\\ P(0) &= 1\end{aligned}$
C: $\begin{aligned}\frac{dP}{dt} &= kP(1-P)\\ P(0) &= 0\end{aligned}$	D: $\begin{aligned}\frac{dP}{dt} &= kP(100,000-P)\\ P(0) &= 0\end{aligned}$

50. Initially, a tank contains 10 litres of pure water. A valve is opened, allowing a brine solution containing 1 gram of salt per liter to enter the tank at a rate of 2 liters per minute. The solution in the tank is stirred constantly, keeping its concentration uniform. The brine drains out of the tank at a constant rate of 2 liters per minute. Let $y(t)$ equal the number of grams of salt in the tank t minutes after the valve is opened. Construct the mathematical model for this situation.

A: $\begin{aligned}\frac{dy}{dt} &= 2+\frac{1}{5}y\\ y(0) &= 0\end{aligned}$	B: $\begin{aligned}\frac{dy}{dt} &= 2+5y\\ y(0) &= 0\end{aligned}$	C: $\begin{aligned}\frac{dy}{dt} &= 2-\frac{1}{5}y\\ y(0) &= 0\end{aligned}$
D: $\begin{aligned}\frac{dy}{dt} &= \frac{2}{5}+y\\ y(0) &= 0\end{aligned}$	E: $\begin{aligned}\frac{dy}{dt} &= \frac{2}{5}-5y\\ y(0) &= 0\end{aligned}$	