Mathematics 030 Final Examination Saturday, April 22, 2006

1. Let $\mathbf{u} = (1, 1, 0, 2)$ and $\mathbf{v} = (3, 2, -1, 1)$. Find $\mathbf{u} + \mathbf{v}$.

A: $(2, 1, -1, -1)$	B: $(-2, -1, 1, 1)$	C: $(3, 2, 0, 2)$	D: $(4, 3, 1, 3)$	E: $(4, 3, -1, 3)$

2. Let $\mathbf{u} = (1, 1, 0, 2)$ and $\mathbf{v} = (3, 2, -1, 1)$. Find $\mathbf{u} \bullet \mathbf{v}$.

A: 7	B: 5	C: 4	D: 20	E: $(3, 2, 0, 2)$

3. Which of the following are linear equations in x, y and z?

(i) $x + \sqrt{y} + 2z = 5$

(ii) $2x + 3y - 4z = \ln 2$

(iii) $x + xy + z = 8$

A: (i) only	B: (ii) only	C: (iii) only	D: none of them	E: all of them

4. Find the augmented matrix for the following system of linear equations.

$$\begin{array}{rcrcrcr} 6x & - & y & + & 5z & = & 1 \\ 3x & + & 2y & + & z & = & 4 \end{array}$$

A: $\begin{bmatrix} 6 & -1 & 5 \\ 3 & 2 & 1 \end{bmatrix}$	B: $\left[\begin{array}{rrr\|r} 6 & -1 & 5 & 0 \\ 3 & 2 & 1 & 0 \end{array}\right]$	C: $\left[\begin{array}{rrr\|r} 6 & -1 & 5 & 1 \\ 3 & 2 & 1 & 4 \end{array}\right]$

5. Find the augmented matrix for the following system of linear equations.

$$\begin{aligned} x - y &= 1 \\ 2y - 3 &= -x \\ x &= 4 \end{aligned}$$

A: $\left[\begin{array}{rr\|r} -1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 4 \end{array}\right]$	B: $\left[\begin{array}{rr\|r} 1 & -1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & 4 \end{array}\right]$	C: $\left[\begin{array}{rr\|r} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & 4 \end{array}\right]$
D: $\left[\begin{array}{rr\|r} -1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & 4 \end{array}\right]$	E: $\left[\begin{array}{rr\|r} 1 & -1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 4 \end{array}\right]$	

6. Which one of the following is not in row-reduced echelon form?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A: A	B: B	C: C	D: D	E: E

7. The system of linear equations

$$\begin{array}{rcrcrcl} x & - & 2y & + & z & = & 2 \\ 2x & - & 4y & + & 2z & = & 4 \\ -x & + & 2y & - & z & = & 2 \end{array}$$

has

A: a three-parameter family of solutions
B: a two-parameter family of solutions
C: a one-parameter family of solutions
D: a unique solution
E: no solution

8. The system of linear equations

$$\begin{array}{rcrcrcl} x & - & y & + & 2z & = & 2 \\ & & -2y & + & 4z & = & 4 \\ & & -y & + & 2z & = & 2 \end{array}$$

has

A: a three-parameter family of solutions
B: a two-parameter family of solutions
C: a one-parameter family of solutions
D: a unique solution
E: no solution

9. Let $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 3 & 6 \end{array}\right]$ be the augmented matrix for a system of linear equations. The solution to this system of equations is

A: $(1,3,2)$	B: $(5,3,2)$	C: $(5,3,6)$	D: $(-1,3,2)$	E: none of A,B,C,D

10. Let $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right]$ be the augmented matrix for a system of linear equations. The solution to this system of equations is

A: $(0,t,1)$	B: $(1-2t,t,1)$	C: $(1-2t,0,t)$	D: $(1-2t,1,t)$	E: $(1+2t,1,t)$

Use the following in questions 11, 12 and 13.

The matrix $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & c^2-1 & c-1 \end{array}\right]$ is the augmented matrix for a system of three linear equations in three unknowns x, y and z.

11. For what value(s) of c does this system have exactly one solution?

A: no values	B: $c=-1$ only	C: $c=1$ only	D: $c=1$ or -1 only	E: $c \neq \pm 1$

12. For what value(s) of c does this system have infinitely many solutions?

A: no values	B: $c=-1$ only	C: $c=1$ only	D: $c=1$ or -1 only	E: $c \neq \pm 1$

13. For what value(s) of c does this system have no solution?

A: no values	B: $c = -1$ only	C: $c = 1$ only	D: $c = 1$ or -1 only	E: $c \neq \pm 1$

14. If A is a 2×3 matrix, B is a 3×3 matrix and C is a 3×2 matrix, which one of the following operations is not defined?

A: $AC + B$	B: BC	C: $A^T + C$	D: CA	E: $C^T B$

15. If $\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = [c_{ij}]$, find c_{21}.

A: 7	B: -7	C: -1	D: 9	E: 11

16. Find the value of x for which $\begin{bmatrix} 5 & x \\ 1 & 4 \end{bmatrix}$ has no inverse.

A: -20	B: -5	C: 0	D: 5	E: 20

17. Let $A = \begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}$. If $A^{-1} = B = [b_{ij}]$, find b_{12}.

A: $-\frac{1}{8}$	B: $\frac{5}{8}$	C: $\frac{1}{8}$	D: $-\frac{3}{8}$	E: 3

18. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. If $A^{-1} = B = [b_{ij}]$, find b_{13}.

A: 4	B: 0	C: -1	D: 2	E: 1

19. The rank of $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 0 \end{bmatrix}$ is

A: 2	B: 0	C: 1	D: 4	E: 3

20. Given $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$. In the solution to the system of linear equations

$$\begin{array}{ccccccc} ax & + & by & + & cz & = & 3 \\ dx & + & ey & + & fz & = & 2 \\ gx & + & hy & + & iz & = & 1 \end{array}$$

the value of x is

A: 1	B: 3	C: 2	D: 0	E: -1

21. A system of linear equations has 5 equations in 8 unknowns. Which one of the following is true?
(A) The system always has a solution.
(B) The system never has a solution.
(C) If the system has a solution then it has exactly 3 parameters in the solution.

(D) If rank $A =$ rank $[A \mid \mathbf{b}] = 3$ then the linear system $A\mathbf{x} = \mathbf{b}$ has exactly 5 parameters in the solution.

A: A	B: B	C: C	D: D	E: none of A,B,C,D

22. Which one of the following is true?
(A) If rank $A =$ rank $[A \mid \mathbf{b}] = 5$, then the corresponding linear system $A\mathbf{x} = \mathbf{b}$ must have exactly 5 equations in exactly 5 unknowns.
(B) If rank $A =$ rank $[A \mid \mathbf{b}] = 5$ and the corresponding linear system $A\mathbf{x} = \mathbf{b}$ has exactly 5 unknowns, then the system has a unique solution.
(C) Every homogeneous linear system of 5 equations in 5 unknowns has only the trivial solution.
(D) Every linear system of 5 equations in 5 unknowns has a unique solution.

A: A	B: B	C: C	D: D	E: none of A,B,C,D

23. If rank $A = 3$ and rank$[A \mid \mathbf{b}] = 4$, then the corresponding linear system $A\mathbf{x} = \mathbf{b}$ has
(i) no solution
(ii) exactly one solution
(iii) exactly three or four solutions
(iv) exactly 1 parameter in the solution.

A: (i)	B: (ii)	C: (iii)	D: (iv)	E: none of A,B,C,D

24. Which one of the following is true?
(A) Every system of linear equations with 7 equations in 4 unknowns has infinitely many solutions.
(B) Every system of linear equations with 4 equations in 7 unknowns has infinitely many solutions.
(C) Every system of homogeneous linear equations with 4 equations in 7 unknowns has infinitely many solutions.
(D) Every system of homogeneous linear equations with 7 equations in 4 unknowns has infinitely many solutions.

A: A	B: B	C: C	D: D	E: none of A,B,C,D

25. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ and B is the row reduced echelon form of A, then the first row of B is

A: $\begin{bmatrix} 1 & 0 & 2 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$	D: $\begin{bmatrix} 1 & 0 & \frac{1}{2} \end{bmatrix}$	E: $\begin{bmatrix} 1 & 0 & -\frac{1}{2} \end{bmatrix}$

26. Find det $\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$.

A: 11	B: 5	C: 10	D: -11	E: -5

27. Find the 2,3-cofactor of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

A: 8	B: 2	C: 16	D: -8	E: -16

28. Find $\det \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

A: 4	B: 6	C: 8	D: -6	E: -4

29. Which one of the following is false?

(i) $\det \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 0$

(ii) $\det \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix} = 8$

(iii) $\det \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 4 & 1 & 2 \end{bmatrix} = 8$

(iv) $\det \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 8$

(v) $\det \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 4 \end{bmatrix} = 8$

A: (i)	B: (ii)	C: (iii)	D: (iv)	E: (v)

30. If $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 5$, then $\det \begin{bmatrix} g & h & i \\ 3d & 3e & 3f \\ a & b & c \end{bmatrix} =$

A: -15	B: 15	C: 5	D: -5	E: none of A,B,C,D

31. If $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 3$, then $\det \begin{bmatrix} a & d & g+2a \\ b & e & h+2b \\ c & f & i+2c \end{bmatrix} =$

A: -3	B: 3	C: -6	D: 6	E: none of A,B,C,D

32. Find $\det \begin{bmatrix} 1 & 2 & 4 & 1 \\ 1 & 4 & 5 & 4 \\ 1 & 2 & 6 & 2 \\ 1 & 2 & 6 & 5 \end{bmatrix}$.

A: 16	B: 15	C: 12	D: 32	E: 18

33. Let A be a 3×3 matrix with $\det A = 5$. Find the value of $\det(2A)$.

A: 250	B: 10	C: 25	D: 40	E: 20

34. Let A and B be $n \times n$ matrices with $\det A = 5$ and $\det B = 10$. Which of the following are false?

(i) A and B are both invertible.
(ii) $\det(A^{-1}B) = 2$
(iii) $\det(B^T A^T) = 50$
(iv) Every linear system $A\mathbf{x} = \mathbf{b}$ has a unique solution.

A: none of them	B: (i) only	C: (ii) only	D: (iii) only	E: (iv) only

35. For the linear system

$$\begin{array}{ccccccc} ax & + & by & + & cz & = & k \\ dx & + & ey & + & fz & = & l \\ gx & + & hy & + & jz & = & m \end{array},$$

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} = 4, \qquad \det\begin{bmatrix} k & b & c \\ l & e & f \\ m & h & j \end{bmatrix} = 8,$$

$$\det\begin{bmatrix} a & k & c \\ d & l & f \\ g & m & j \end{bmatrix} = 12, \qquad \det\begin{bmatrix} a & b & k \\ d & e & l \\ g & h & m \end{bmatrix} = 20.$$

Then, in the solution for this system, the value of z is

A: 2	B: 4	C: 8	D: 20	E: 5

36. A is a 3×3 matrix with $\det A = 4$ and Adj $A = \begin{bmatrix} -4 & 5 & -2 \\ -4 & 2 & 0 \\ 4 & -3 & 2 \end{bmatrix}$. If $A^{-1} = [c_{ij}]$, then the value of c_{21} is

A: -4	B: -1	C: -16	D: 4	E: 1

37. If A is a 4×4 matrix with $\det(A) = 3$, then $\det(\text{Adj } A) =$

A: $\frac{1}{3}$	B: 3	C: 9	D: 27	E: 81

38. Let $\mathbf{u} = (2, 1, 3), \mathbf{v} = (-3, 2, 4)$ and $\mathbf{w} = (3, 12, 30)$. Then $\mathbf{w}$ can be written (uniquely) as a linear combination $a\mathbf{u} + b\mathbf{v}$. Find the value of a.

A: 3	B: 4	C: 5	D: 6	E: 7

39. Let $\mathbf{u} = (1, 1, 1), \mathbf{v} = (0, 2, 1)$. Which one of the following is <u>not</u> a linear combination of $\mathbf{u}$ and $\mathbf{v}$?

A: $(0, 0, 0)$	B: $(1, 1, 1)$	C: $(1, 3, 2)$	D: $(1, -1, 0)$	E: $(2, 4, 2)$

40. Which one of the following sets of vectors is <u>linearly independent</u>?

A: $\{(1, 0, 0), (1, 2, 0), (2, 3, 5)\}$	B: $\{(1, 1, 1), (1, 0, 1), (2, 2, 2)\}$
C: $\{(1, 0, 0), (0, 1, 0), (1, 1, 0)\}$	D: $\{(1, 1, 1), (1, 1, 0), (0, 1, 1), (1, 0, 1)\}$
E: $\{(0, 0, 0)\}$	

41. Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ be vectors in $\mathbb{R}^4$ and let A be the 4×5 matrix with i^{th} column $\mathbf{a}^i = \mathbf{v}_i$. Suppose that the row-reduced echelon form of A is

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which one of the following is <u>false</u>?

A: $\mathbf{v}_2 = 2\mathbf{v}_1$	B: $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.
C: $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.	D: $\mathbf{v}_4 = -\mathbf{v}_1 + 2\mathbf{v}_3$
E: $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_5\}$ is linearly independent.	

42. For what value of k is the set of vectors $\{(1,0,0),(1,2,8),(1,3,k)\}$ linearly dependent?

A: 8	B: 16	C: 12	D: 4	E: 24

43. Which one of the following sets of vectors is a subspace of $\mathbb{R}^3$?

A: $\{(0,0,0),(1,1,1)\}$
B: $\{(a, a+1, a) \mid a$ is a real number$\}$
C: $\{(a,b,c) \mid a^2+b^2+c^2 = 16\}$
D: the set of all vectors in $\mathbb{R}^3$ except the vector $(1,0,1)$
E: $\{(a, a+b, b) \mid a, b$ are real numbers$\}$

44. Which one of the following sets of vectors is not a subspace of $\mathbb{R}^3$?

A: $\{(0,0,0)\}$
B: $\{a(1,1,1) \mid a \geq -2\}$
C: $\mathbb{R}^3$
D: $\{(a,b,c) \mid 2a+3b-4c = 0\}$
E: the set of all linear combinations of the vectors $(1,1,1)$ and $(2,2,2)$.

45. Which one of the following sets of vectors is a spanning set for $\mathbb{R}^3$?

A: $\{(1,1,1),(1,0,1)\}$	B: $\{(1,1,1),(2,2,2),(3,3,3),(1,1,0)\}$
C: $\{(1,0,0),(0,1,0),(0,0,1),(1,1,1)\}$	D: $\{(1,0,0),(0,1,0),(1,1,0)\}$
E: $\{(0,0,0),(1,0,1),(1,1,0)\}$	

46. Let S be the subspace of $\mathbb{R}^4$ of all vectors of the form $(x+y, x-y, x, y)$. Which one of the following is false?

A: The vector $(0,0,0,0)$ is in S.
B: The vector $(2,0,1,1)$ is in S.
C: $\{(1,1,1,0),(1,-1,0,1)\}$ is a basis for S.
D: $\{(2,0,1,1),(1,-1,0,1)\}$ is a basis for S.
E: The dimension of S is 4.

47. Which one of the following is false?

A: $\{(0,0,0)\}$ is a subspace of $\mathbb{R}^3$ with dimension 1.
B: The dimension of $\mathbb{R}^3$ is 3.
C: Every set of 4 vectors in $\mathbb{R}^3$ is linearly dependent.
D: $\{(1,1,1)\}$ can be extended to a basis for $\mathbb{R}^3$.
E: The subspace of $\mathbb{R}^3$ spanned by the set $\{(0,0,0),(1,0,1),(0,1,0),(1,1,1)\}$ has dimension 2.

48. The matrix $\begin{bmatrix} 1 & 2 & 2 & 0 & 3 \\ 0 & 1 & 0 & 1 & -2 \\ 1 & 2 & 2 & 0 & 3 \\ 0 & 1 & 0 & 1 & -2 \end{bmatrix}$ has row-reduced echelon form $\begin{bmatrix} 1 & 0 & 2 & -2 & 7 \\ 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Let S be the subspace of $\mathbb{R}^4$ spanned by $\{(1,0,1,0),(2,1,2,1),(2,0,2,0),(0,1,0,1),(3,-2,3,-2)\}$. A basis for S is

A: $\{(1,0,0,0),(0,1,0,0)\}$	B: $\{(2,0,0,0),(-2,1,0,0),(7,-2,0,0)\}$
C: $\{(1,0,1,0),(2,1,2,1)\}$	D: $\{(2,0,2,0),(0,1,0,1),(3,-2,3,-2)\}$
E: none of the above	

49. Consider the homogeneous system of linear equations

$$\begin{array}{rcrcrcrcl} x_1 & + & x_2 & + & x_3 & - & x_4 & = & 0 \\ x_1 & + & 2x_2 & + & x_3 & & & = & 0 \\ 2x_1 & + & 3x_2 & + & 2x_3 & - & x_4 & = & 0 \end{array}.$$

The augmented matrix for the system has row-reduced echelon form

$$\left[\begin{array}{cccr|c} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

so a basis for the solution space of this system is

A: $\{(1,0,1,-2),(0,1,0,1)\}$	B: $\{(1,0,1,0),(-2,1,0,1)\}$
C: $\{(1,0,0,0),(0,1,0,0)\}$	D: $\{(-1,0,1,0),(2,-1,0,1)\}$
E: none of the above	

50. The matrix $\begin{bmatrix} 1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

has row-reduced echelon form $\begin{bmatrix} 1 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$.

Therefore, a basis for $\mathbb{R}^5$ which includes the vectors $\mathbf{v}_1 = (1,1,1,1,1)$, $\mathbf{v}_2 = (1,-1,-1,-1,-1)$ and $\mathbf{v}_3 = (-1,-1,-1,-1,1)$ is

A: $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,(0,0,0,1,0),(0,0,0,0,1)\}$
B: $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,(0,1,0,0,0),(0,0,1,0,0)\}$
C: $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,(1,0,0,0,0),(0,0,0,0,1)\}$
D: $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,(1,0,0,0,0),(0,1,0,0,0),(0,0,1,0,0)\}$
E: none of the above.