Mathematics 030 Final Examination Wednesday, April 13, 2005

1. If $\mathbf{u} = (3, -1, 1, 2)$ and $\mathbf{v} = (2, -2, 4, 1)$, find $2\mathbf{u} - 3\mathbf{v}$.

A: $(12, 4, 14, 5)$	B: $(-5, -1, 5, -4)$	C: $(6, 2, 4, 2)$
D: $(-36, -12, -24, -12)$	E: $(0, 4, -10, 1)$	

2. If $\mathbf{u} = (3, -1, 1, 2)$ and $\mathbf{v} = (2, -2, 4, 1)$, find $\mathbf{u} \bullet \mathbf{v}$.

A: -5	B: 14	C: -84	D: 35	E: none of A,B,C,D

3. Which of the following systems of equations are linear?

(i) $\begin{aligned} \ln x - 2y + z &= 0 \\ x + 5y - z &= 3 \end{aligned}$

(ii) $\begin{aligned} x + \frac{y}{5} &= 2 \\ \frac{x}{3} + y &= 7 \end{aligned}$

(iii) $\begin{aligned} x + \frac{5}{y} &= 2 \\ \frac{3}{x} + y &= 7 \end{aligned}$

(iv) $\begin{aligned} x^2 + xy - z^2 &= 1 \\ x - y^2 + z^2 &= 2 \end{aligned}$

A: all of them	B: none of them	C: (i), (ii), (iii) only
D: (ii) and (iii) only	E: (ii) only	

4. Find the augmented matrix for the system

$$\begin{aligned} x_1 &= x_2 - x_3 \\ x_2 + 3 &= x_4 \\ x_3 + x_4 &= 7 \end{aligned}$$

A: $\left[\begin{array}{cccc|c} 0 & 1 & 2 & -1 & 0 \\ 2 & 3 & 4 & 0 & 0 \\ 3 & 4 & 7 & 0 & 0 \end{array}\right]$	B: $\left[\begin{array}{cccc|c} 0 & 1 & 1 & -1 & 0 \\ 1 & 3 & 1 & 0 & 0 \\ 1 & 1 & 7 & 0 & 0 \end{array}\right]$	C: $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 1 & 7 \end{array}\right]$
D: $\left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -3 \\ 0 & 0 & 1 & 1 & 7 \end{array}\right]$	E: $\left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$	

5. Which of the following matrices are in row-reduced echelon form?

(i) $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(vi) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A: (ii) and (vi) only	B: (iii) and (v) only	C: (i) and (iv) only
D: (iv) only	E: (iii) only	

6. The given matrix is the augmented matrix for a linear system in the variables x_1, x_2 and x_3. Solve the system.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

A: no solution	B: $(-1, 1, -1)$	C: $(-1 - s + t, s, t)$
D: $(-1 + s - t, s, t)$	E: $(1 - s + t, s, t)$	

7. The given matrix is the augmented matrix for a linear system in the variables x_1, x_2 and x_3. Solve the system.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 1 \\ -2 & 1 & -1 & 0 \\ 0 & 3 & -3 & 3 \end{array}\right]$$

A: no solution	B: $(1, 0, 3)$	C: $(0, t, t)$	D: $(0, -t, t)$	E: $(0, s, t)$

8. The given matrix is the augmented matrix for a linear system in the variables x_1, x_2 and x_3. Solve the system.

$$\left[\begin{array}{rrr|r} 1 & 0 & -1 & -1 \\ 1 & 1 & 1 & 4 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

A: no solution	B: $(-1, 4, 1)$	C: $(-1, 5, 3)$	D: $(2, -1, 3)$	E: $(-1, s, t)$

9. Find the $(3, 4)$-entry of the row-reduced echelon form of

$$A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 2 & 3 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

A: 0	B: 1	C: 5	D: 10	E: 15

10. The augmented matrix of a system of linear equations has row-reduced echelon form $\left[\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$. The solution is

A: $(0,4,0)$	B: $(0,-3s,-2,t)$	C: $(3s-t,s,t,t-2)$
D: $(-3s+t,s,t,t-2)$	E: $(3s-t,s,4+2t,t)$	

11. For which value(s) of c does the system

$$\begin{array}{rcrcrcl} x & & & - & 2z & = & 1 \\ x & + & cy & + & 8z & = & 0 \\ x & + & y & & & = & 1 \end{array}$$

have a unique solution?

A: $c=5$ only	B: $c\neq 5$	C: $c=-5$ only	D: $c\neq -5$	E: no values

12. For which value(s) of c does the system

$$\begin{array}{rcrcrcl} x & - & 2y & + & z & = & 0 \\ & & y & - & z & = & 0 \\ 2x & + & y & + & cz & = & 0 \end{array}$$

have infinitely many solutions?

A: $c=-3$ only	B: $c\neq -3$	C: $c=3$ only	D: $c\neq 3$	E: no values

For questions 13, 14, 15 and 16 let

$$A=\begin{bmatrix} 2 & 5 & -2 \\ 3 & -1 & 1 \end{bmatrix},\quad B=\begin{bmatrix} 1 & 3 & 1 \\ -5 & 0 & 8 \end{bmatrix} \text{ and } C=\begin{bmatrix} 0 & 3 & 2 & 1 \\ 2 & 1 & -2 & 4 \\ -3 & 4 & -1 & 0 \end{bmatrix}.$$

13. Find the $(2,1)$-entry of $A-2B$.

A: 0	B: -1	C: 13	D: -11	E: undefined

14. Find the $(3,1)$-entry of A^T.

A: -2	B: 1	C: 8	D: 15	E: undefined

15. Find the $(2,3)$-entry of AC.

A: -4	B: 7	C: -3	D: 6	E: undefined

16. Find the $(2,4)$-entry of B^TC.

A: 0	B: 4	C: -5	D: 1	E: undefined

17. Find A^2 where A is the matrix $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$.

A: $\begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix}$	B: $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$	C: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	D: $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	E: $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

18. Let $A = \begin{bmatrix} 3 & -4 \\ -2 & 6 \end{bmatrix}$. If $A^{-1} = B = [b_{ij}]$, find b_{22}.

A: $\frac{2}{10}$	B: $\frac{3}{10}$	C: $\frac{4}{10}$	D: $\frac{6}{10}$	E: none of A,B,C,D

19. Find the value of x for which $\begin{bmatrix} 3 & 2 \\ 6 & x \end{bmatrix}$ has no inverse.

A: 2	B: 3	C: 4	D: 5	E: 6

20. Let $A = \begin{bmatrix} 1 & 0 & -2 \\ -1 & 1 & 2 \\ 1 & -2 & 0 \end{bmatrix}$. If $A^{-1} = B = [b_{ij}]$, find b_{12}.

A: 0	B: 1	C: -1	D: 2	E: $\frac{1}{2}$

21. Given $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & 1 \\ -3 & 2 & -1 \end{bmatrix}$. In the solution to the system of linear equations

$$\begin{array}{rcrcrcr} ax & + & by & + & cz & = & -1 \\ dx & + & ey & + & fz & = & 7 \\ gx & + & hy & + & iz & = & 2 \end{array}$$

the value of z is

A: 2	B: -3	C: 4	D: 15	E: -16

22. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 3 & -2 \\ 3 & 1 & 0 \end{bmatrix}$.

A: 0	B: 1	C: 2	D: 3	E: 4

23. Which one of the following is true?

(A) Every homogeneous linear system of 5 equations in 9 unknowns has infinitely many solutions.

(B) Every homogeneous linear system of 9 equations in 5 unknowns has infinitely many solutions.

(C) Every linear system of 5 equations in 9 unknowns has infinitely many solutions.

(D) Every linear system of 9 equations in 5 unknowns has infinitely many solutions.

A: (A)	B: (B)	C: (C)	D: (D)	E: none of (A),(B),(C),(D)

24. If A is a 6×8 matrix of rank 3, then every linear system with coefficient matrix A which has a solution has

A: a 1-parameter family of solutions	B: a 2-parameter family of solutions
C: a 3-parameter family of solutions	D: a 4-parameter family of solutions
E: a 5-parameter family of solutions	

25. Which one of the following is true?

(A) If a linear system of 4 equations in 4 unknowns has no solutions, its coefficient matrix must be invertible.

(B) If a linear system with 4 equations has no solutions, the rank of its coefficient matrix must be less than 4.

(C) If a linear system with 4 equations has no solutions, the rank of its coefficient matrix must be equal to 4.

(D) If a linear system with 4 equations has no solutions, the rank of its coefficient matrix must be greater than 4.

A: (A)	B: (B)	C: (C)	D: (D)	E: none of (A), (B), (C), (D)

26. Find det $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

A: -2	B: -1	C: 0	D: 1	E: 2

27. Find det $\begin{bmatrix} 1 & \sqrt{2} & 1 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

A: 6	B: $\sqrt{2}-2$	C: $\sqrt{2}+2$	D: -2	E: -6

28. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find the submatrix A_{23}.

A: $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$	B: $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$	C: $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$	D: $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$	E: $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

29. Find det $\begin{bmatrix} 0 & 1 & 1 & -3 \\ 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -1 & 1 \end{bmatrix}$.

A: -4	B: 4	C: -2	D: 2	E: -1

30. If det $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 4$ then det $\begin{bmatrix} 2a & d & g \\ 2b & e & h \\ 2c & f & i \end{bmatrix} =$

A: 4	B: 32	C: 8	D: -4	E: -32

31. If det $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 2$ then det $\begin{bmatrix} a-d & b-e & c-f \\ 2g & 2h & 2i \\ d & e & f \end{bmatrix} =$

A: 2	B: 4	C: -2	D: -4	E: 0

32. Find det $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 3 & 0 \\ -1 & 2 & -2 & 0 \end{bmatrix}$.

A: 14	B: 12	C: 10	D: 8	E: 6

33. Let A and B be 4×4 matrices with $\det A = 2$ and $\det B = 4$. Which of the following statements are true?
(i) $\det(AB) = \det(BA)$
(ii) $\det(2A) = 32$
(iii) $\det(A^{-1}B) = 2$

A: none of them	B: all of them	C: (i) only	D: (ii) only	E: (iii) only

34. Suppose A is an $n \times n$ matrix with $\det(A) = 0$. Which one of the following statements is false?
(i) Every system $A\mathbf{x} = \mathbf{b}$ has a unique solution.
(ii) $\text{rank}(A) < n$
(iii) $\det(A^T) = 0$
(iv) The system $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions.
(v) A is not invertible.

A: (i)	B: (ii)	C: (iii)	D: (iv)	E: (v)

35. Consider the system

$$\begin{array}{rcrcl} ax & + & by & = & m \\ cx & + & dy & = & n. \end{array}$$

Suppose $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = -1$, $\det \begin{bmatrix} a & m \\ c & n \end{bmatrix} = -2$ and $\det \begin{bmatrix} m & b \\ n & d \end{bmatrix} = 3$. Then the value of y is

A: 2	B: -2	C: 3	D: -3	E: -1

36. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Then $\text{Adj}(A) =$

A: $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$	B: $\begin{bmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{bmatrix}$	C: $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$	D: $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$	E: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

37. If A is a 3×3 matrix with $\det(A) = 2$, then $\det(\text{Adj}\, A) =$

A: 2	B: 4	C: 8	D: 16	E: 0

38. Suppose $\mathbf{u} = (3, 2, 4)$, $\mathbf{v} = (0, 1, 2)$ and $\mathbf{w} = (1, 0, 0)$. Then $\mathbf{u}$ can be written as $\mathbf{u} = a\mathbf{v} + b\mathbf{w}$. Find the value of a.

A: 2	B: 3	C: 4	D: -3	E: 0

39. Which of the following vectors is <u>not</u> a linear combination of $\mathbf{v} = (0, 1, 2)$ and $\mathbf{w} = (1, 0, 0)$?

A: $(7, 1, 2)$	B: $(0, -1, -2)$	C: $(0, 0, 0)$	D: $(0, 1, 1)$	E: $(0, 1, 2)$

40. For which value of k is $(0, 1, k)$ a linear combination of $(1, 1, 1)$ and $(2, 3, 1)$?

A: -2	B: -1	C: 0	D: 1	E: 2

41. Which one of the following sets of vectors spans $\mathbb{R}^3$?

A: $\{(1,1,1)\}$	B: $\{(1,1,1),(2,2,2),(3,3,3)\}$
C: $\{(1,0,0),(1,1,0)\}$	D: $\{(1,0,0),(1,1,0),(1,1,1),(0,0,0)\}$
E: $\{(0,0,0),(1,0,0),(0,1,0)\}$	

42. Which one of the following sets of vectors is linearly independent?

A: $\{(1,1,1),(1,2,3),(2,3,4)\}$	B: $\{(0,0,0),(1,0,0),(0,1,0)\}$
C: $\{(1,2,1),(0,1,0),(1,3,-1),(7,1,3)\}$	D: $\{(1,1,0),(1,1,1)\}$
E: $\{(1,2,0),(2,3,0),(-1,4,0)\}$	

43. Which one of the following sets of vectors is linearly dependent?

A: $\{(1,2,1)\}$	B: $\{(1,0,0),(0,1,0)\}$	C: $\{(1,0,1),(1,1,0),(2,1,1)\}$
D: $\{(1,0,0),(1,1,0),(1,1,1)\}$	E: none of A,B,C,D	

44. Which one of the following sets of vectors is a subspace of $\mathbb{R}^3$?

A: $\{(a,b,c) \mid a+b+c=0\}$	B: $\{(a,b,c) \mid abc=0\}$	C: $\{(a,b,c) \mid a+b+c=1\}$
D: $\{(a,b,c) \mid abc=1\}$	E: $\{(a,b,c) \mid a+b+c \geq 0\}$	

45. Which one of the following sets of vectors is <u>not</u> a subspace of $\mathbb{R}^3$?

A: $\{(a,b,c) \mid a+b-c=0\}$
B: $\{(1,0,0),(0,1,0),(0,0,1)\}$
C: $\{(a,b,c) \mid a=0\}$
D: The set of all solutions of $A\mathbf{x}=\mathbf{0}$, where A is a 5×3 matrix.
E: The set of all linear combinations of the vectors $(1,1,-1)$ and $(2,1,0)$.

46. Let S be a subspace of $\mathbb{R}^4$ with $\dim S = 2$. Suppose A is a 4×4 matrix with $S = \{\mathbf{x}\in\mathbb{R}^4 \mid A\mathbf{x}=\mathbf{0}\}$. Which one of the following statements is false?

A: Every basis of S contains exactly two vectors.
B: $\text{rank}(A)=2$
C: A is invertible.
D: Any basis of S can be extended to a basis of $\mathbb{R}^4$.
E: S is closed under addition.

47. Let A be a 6×7 matrix of rank 3. Then the dimension of the solution space of the homogeneous system $A\mathbf{x}=\mathbf{0}$ is

A: 0	B: 1	C: 2	D: 3	E: 4

48. The matrix $\begin{bmatrix} 1 & -2 & 0 & 1 & 0 \\ 2 & -4 & 1 & 1 & 0 \\ -1 & 2 & 0 & -1 & 1 \end{bmatrix}$ has row-reduced echelon form $\begin{bmatrix} 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.
Let S be the subspace of $\mathbb{R}^3$ spanned by $\{(1,2,-1),(-2,-4,2),(0,1,0),(1,1,-1),(0,0,1)\}$. A basis for S is

A: $\{(1,2,-1),(0,1,0),(0,0,1)\}$	B: $\{(1,2,-1),(-2,-4,2),(0,0,1)\}$
C: $\{(1,2,-1),(0,1,0),(1,-1,-1)\}$	D: $\{(-2,0,2),(0,1,0),(1,-1,-1)\}$
E: none of the above	

49. The matrix $\begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ has row-reduced echelon form $\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$.

Therefore, a basis for $\mathbb{R}^4$ which includes the vectors $\mathbf{u} = (1, 1, -1, 1)$ and $\mathbf{v} = (2, 1, -1, 1)$ is

A: $\{\mathbf{u}, \mathbf{v}, (1, 0, 0, 0), (0, 1, 0, 0)\}$	B: $\{\mathbf{u}, \mathbf{v}, (1, 0, 0, 0), (0, 0, 1, 0)\}$
C: $\{\mathbf{u}, \mathbf{v}, (0, 1, 0, 0), (0, 0, 1, 0)\}$	D: $\{\mathbf{u}, \mathbf{v}, (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$
E: none of the above	

50. A certain homogeneous system of linear equations $A\mathbf{x} = 0$ has augmented matrix $\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$. A basis for the solution space of this system is

A: $\{(-1, 2, 1, 0, 0), (-1, -1, 0, 1, 0)\}$	B: $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$
C: $\{(1, -2, 0), (0, 0, 1)\}$	D: $\{(1, 0, 1, 1, 0), (0, 1, -2, 1, 0)\}$
E: $\{(1, 0, 0, 0, 0), (0, 1, 0, 0, 0), (0, 0, 0, 0, 1)\}$	