

SECTION B

7. State the condition for the wave function of a particle moving in one dimension to be correctly normalized. [2]

At a certain instant a particle has the wave function

$$\psi(x) = \begin{cases} C(a^2 - x^2) & |x| \leq a; \\ 0 & |x| > a. \end{cases}$$

Find a suitable value of the constant C so that ψ obeys the normalization condition. [4]

What is the probability that the particle is found between $x = 0$ and $x = a$? [4]

The expectation value of an operator \hat{O} in one dimension can be written

$$\langle \hat{O} \rangle = \int_{-\infty}^{\infty} \psi^*(x) \hat{O} \psi(x) dx.$$

What is the physical significance of the expectation value? [2]

Calculate the expectation values of the following quantities, using the normalized wave function $\psi(x)$. You may wish to make use of symmetry arguments where possible to simplify the working.

- (a) The position \hat{x} ; [2]
- (b) The momentum \hat{p} ; [2]
- (c) The kinetic energy $\hat{p}^2/2m$, where m is the particle's mass. [4]