UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics B3C: Pure Mathematics

COURSE CODE : **MATHB03C**

UNIT VALUE : **0.50**

DATE : **05-MAY-06**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

06-C0997-3-50

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Evaluate the indefinite integrals

$$\text{(i)} \int \frac{x - 3x^2 + 1}{x^2}\,\mathrm{d}x, \qquad \text{(ii)} \int (x-1)^2(x+1)\,\mathrm{d}x, \qquad \text{(iii)} \int \frac{\log x}{x}\,\mathrm{d}x.$$

(b) By using the substitution $t = \tan\left(\frac{x}{2}\right)$ or otherwise find

$$\int \frac{\mathrm{d}x}{3\sin x + 4\cos x}.$$

2. (a) Find all the eigenvalues of each of the matrices

$$\text{(i)} \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}, \quad \text{(ii)} \begin{pmatrix} -1 & -5 & 2 \\ -3 & -3 & 2 \\ -4 & -16 & 8 \end{pmatrix}.$$

(b) For the matrix in part (ii) of (a), find an eigenvector associated with each eigenvalue.

3. (a) By expressing $\dfrac{1}{k(k+1)}$ as the difference of two fractions, or otherwise, evaluate the sum

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)}.$$

(b) Let $f(x)$ be a function. Write down the Maclaurin series expansion for $f(x)$.

(c) For $|x| < 1$ find the Maclaurin series expansions for up to and including x^5 for the functions

$$\text{(i) } (1+x)^{-2}, \ \text{(ii) } (1+x)^{\frac{1}{2}}.$$

4. (a) Let $i = \sqrt{-1}$. Write the following complex numbers in the form $re^{i\theta}$:

$$\text{(i) } 1+i, \quad \text{(ii) } \frac{1}{1+i}, \quad \text{(iii) } (\sqrt{3}-i)^{\frac{1}{2}}.$$

(b) Find all solutions of

$$z^8 = 1$$

in the form $z = x + iy$ and plot them on the Argand diagram.

5. Consider the function

$$y(x) = x + \frac{1-x}{1+x}.$$

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

(b) Find the turning points of y and determine whether they are maxima, minima or points of inflexion.

(c) Find all asymptotes of the curve $y(x)$.

(d) Sketch the function $y(x)$ being careful to show any turning points and asymptotes.

6. (a) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{2y^2 + yx}{x^2}$$

that satisfies $y(1) = 1$.

(b) Find the general solution of the differential equation

$$(1+x)\frac{dy}{dx} + xy = e^{-x}.$$

7. (a) Find the determinants of each of the following matrices

$$\text{(i)} \begin{pmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{pmatrix}, \quad \text{(ii)} \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}, \quad \text{(iii)} \begin{pmatrix} 2 & -4 & 2 & 2 \\ 0 & 5 & 1 & 1 \\ 2 & -1 & 1 & 0 \\ -1 & 0 & 6 & -10 \end{pmatrix}.$$

(b) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}.$$

END OF PAPER