# UNIVERSITY COLLEGE LONDON

University of London

## EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

*B.Sc.* *LL.B.* *M.Sci.*

**Mathematics B3C: Pure Mathematics**

| | | |
|---|---|---|
| COURSE CODE | : | **MATHB03C** |
| UNIT VALUE | : | **0.50** |
| DATE | : | **12-MAY-04** |
| TIME | : | **14.30** |
| TIME ALLOWED | : | **2 Hours** |

04-C0908-3-60


**TURN OVER**

*All questions may be attempted but only marks obtained on the best* **five** *solutions will count.*
*The use of an electronic calculator is* **not** *permitted in this examination.*

1. (a) Evaluate the indefinite integrals

$$\text{(i)} \int \frac{x+5}{1+x^2}\,\mathrm{d}x, \qquad \text{(ii)} \int x^2 \log x\,\mathrm{d}x.$$

(b) Let $I_n = \int_0^\pi e^x \sin(nx)\,\mathrm{d}x$ for integer $n$.
Using integration by parts, or otherwise, find an expression for $I_n$ in terms of $n$.
Show that $I_{10} = \dfrac{10(1-e^\pi)}{101}$.

2. (a) If the complex number $z = x + iy$ ($x, y$ real numbers, $i = \sqrt{-1}$), write down expressions in $x, y$ for

$$\text{(i) } |z|, \quad \text{(ii) } \arg(z).$$

Indicate on the Argand diagram how you choose the angle in part (ii).

(b) Let $z_1 = -1 + i$, $z_2 = \sqrt{3} + i$. Find $|z_1|$, $|z_2|$, $\arg(z_1)$, $\arg(z_2)$. Plot $z_1, z_2$ on the Argand diagram.

(c) What is the polar form for a complex number. Can every complex number be written in polar form?

(d) Show that if $z, w$ are complex numbers, then $|zw| = |z||w|$ and $\arg(zw) = \arg(z) + \arg(w)$. Hence show that $|z^n| = |z|^n$ and that $\arg(z^n) = n\arg(z)$.

(e) Plot all solutions of $z^4 = -1$ on the Argand diagram.

3. Consider the function

$$y(x) = \frac{x^2+3}{x+1}.$$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Find the turning points of $y$ and determine whether they are maxima, minima or points of inflexion.

(c) Find all asymptotes of the curve $y(x)$.

(d) Sketch the function $y(x)$ being careful to show any turning points and asymptotes.

4. (a) Find the general solution of the differential equation

$$x\frac{dy}{dx} = y + xy.$$

Find the solution that satisfies $y(1) = 2$.

(b) Find the general solution of the differential equation

$$\frac{dy}{dx} - y\tan x = \cos x.$$

5. (a) Find the determinants of the following matrices

$$\text{(i)} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix}, \text{(ii)} \begin{pmatrix} 1 & 2 & 0 & 3 \\ 4 & 2 & 1 & 0 \\ 0 & 1 & 2 & -3 \\ 0 & 3 & -2 & 1 \end{pmatrix}.$$

(b) Find the solution of the system

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 3y &= 2 \\ y + 2z &= -1. \end{aligned}$$

6. (a) Using the Ratio Test, the Comparison Test, or otherwise, establish which of the following series converge:

$$\text{(i)} \sum_{n=1}^{\infty} \frac{1}{n(n+1)}, \quad \text{(ii)} \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}.$$

(b) Suppose that $f(x)$ has the Maclaurin series expansion $\sum_{k=0}^{\infty} a_k x^k$ (which you may assume is valid for all $x$). Show that

$$a_k = \frac{f^{(k)}(0)}{k!}.$$

Find the Maclaurin series expansions up to and including $x^5$ for the functions

$$\text{(i)} \sinh x, \quad \text{(ii)} (1-x)^{\frac{1}{2}}.$$

7. (a) Let $A = \begin{pmatrix} 0 & 3 \\ -1 & 5 \\ 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -1 & 1 \\ -2 & 3 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -3 & -6 \end{pmatrix}$.

Find all possible ways of multiplying 2 of these 3 matrices together and evaluate the result of each multiplication.

(b) Let $A$ be an $n \times n$ matrix. Explain what is meant by an eigenvalue and eigenvector of $A$, and write down the characteristic polynomial that determines the eigenvalues of $A$.

(c) Find all the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & -6 & 6 \\ -2 & 5 & -3 \\ -2 & 6 & -4 \end{pmatrix}.$$

(d) Find an eigenvector for the largest eigenvalue of $A$.



END OF PAPER