UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Mathematics B3C: Pure Mathematics

COURSE CODE	:	**MATHB03C**
UNIT VALUE	:	**0.50**
DATE	:	**15-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0899-3-60

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Sketch the graph of the function

$$y = \frac{x^2}{1 - x^2}$$

clearly indicating intervals of monotonicity, horizontal and vertical asymptotes and approaches to them.

2. (a) What is meant by the Taylor-Maclaurin expansion of a function about $x = 0$?
   (b) Use the binomial theorem to find the expansion of

$$\frac{1}{\sqrt{1 + x^2}}$$

as far as the term in x^6.
   (c) Using L'Hôpital's rule or otherwise evaluate

(i) $\lim_{x \to \pi/2} \frac{2x - \pi}{\cos x}$ (ii) $\lim_{x \to 0} \frac{\arctan(x) - x}{x^3}$ (iii) $\lim_{x \to \infty} \frac{\ln x}{x}$

3. Express

$$\frac{3t^2 + 7t + 4}{(t + 3)(t^2 + 1)}$$

in the form

$$\frac{A}{t + 3} + \frac{Bt + C}{t^2 + 1}$$

for suitable constants A, B, C.
Find the integral

$$\int_0^1 \frac{3t^2 + 7t + 4}{(t + 3)(t^2 + 1)} dt.$$

4. (a) State de Moivre's Theorem and use it to express $(1+i\sqrt{3})^{14}$ in the form $a+ib$.

   (b) Determine the solutions to the equation $z^6 = 1$ and sketch them in the Argand Diagram.

5. (a) Find constants A, B so that $\frac{1}{n^2-1} = \frac{A}{n-1} + \frac{B}{n+1}$, use this to simplify the partial sums $s_N = \sum_{n=2}^{N} \frac{1}{n^2-1}$ and so find

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1}$$

   (b) Find the precise range of the values for which the series

$$\sum_{n=1}^{\infty} \frac{x^n}{4^n n}$$

   converges.

6. (a) Find the inverse of the matrix

$$\begin{pmatrix} 2 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

   (b) Find the general solution to the system of equations

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & - & x_3 & + & x_4 & = & 2 \\ 3x_1 & + & 7x_2 & - & x_3 & + & 2x_4 & = & 6 \\ x_1 & + & x_2 & - & 3x_3 & + & 5x_4 & = & -4 \\ x_1 & + & 4x_2 & + & 3x_3 & + & 2x_4 & = & -4 \end{array}$$

7. (a) Find the general solution to the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = \cos(x)$$

   Find the explicit solution which takes value zero when $x = \pi$.

   (b) A radioactive material has a half-life of 7 years. If 1 kilogram of this material was placed in a safe on 1 January 1975, how much will be left at the end of this year?

MATHB03C END OF PAPER