UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M310: Operational Research and Mathematical Modelling

COURSE CODE	:	**MATHM310**
UNIT VALUE	:	**0.50**
DATE	:	**23-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

05-C0973-3-40

TURN OVER

All questions may be attempted, but only marks obtained in the best four solutions will count. The use of an electronic calculator is not permitted in this examination.

1 Suppose $f: R^2 \to R^1$ is given by $f(x,y) = x^2 + y^2 + 2x + 4y$.

(i) Using results stated in the course, or otherwise, show that f is a convex function.

(ii) Carry out one iteration of the method of steepest descent for minimising f using the origin as the initial point.

(iii) Show that the absolute minimum of f is attained when $x = -1$ and $y = -2$.

2 (a) A Markov chain has n states ($n > 1$) and states 1 to n-1 all communicate with state n. If v_i denotes the expected number of steps after which the chain first reaches state n starting from state i, show that

$$(\mathrm{I} - \mathrm{Q})\, v = \mathbf{1}$$

where Q is the n-1 $\times$ n-1 matrix of transitions between the states 1 to n-1, I is the n-1 $\times$ n-1 identity matrix, $\mathbf{1}^{\mathrm{T}} = (1, 1, 1 \,.....1)$ and $v^{\mathrm{T}} = (v_1, v_2 \,....\, v_{n-1})$.

(b) There are three brands of washing powder on the market, A, B and C. Market researchers use a Markov chain to model how customers change brands with successive purchases. They estimate that customers buying brand A will buy brand A for their next purchase with probability p. Similarly, for brands B and C customers will buy the same brand for their next purchase with probabilities q and r respectively. If a customer changes to another brand, there is an equal probability of choosing either one of the other two.

(i) Write down a transition matrix for this Markov chain.

(ii) If a customer is observed buying brand A, how many more purchases is he expected to make before he buys brand C?

3 (a) The arrivals to a queue are governed by a Poisson process. Show that the distribution of inter-arrival times is exponential.

(b) A new antibiotic is undergoing clinical trials. A single oral dose of the antibiotic is given to a patient. The drug is absorbed via the gastrointestinal (GI) tract into the bloodstream, and then eliminated from the bloodstream. Both these processes are known to be first order with rates k and γ respectively.

(i) Show that the change in plasma concentration, c, of the drug with time, t, obeys the following equation, where A is a constant:

$$c = A\frac{k}{k-\gamma}(e^{-\gamma t} - e^{-kt})$$

Suppose instead that the patient is given an oral dose at time $t = 0$, and a second oral dose at time $t = 2T$, where T is the time at which drug plasma concentration from the first dose reaches a maximum. The pharmacokinetic characteristics of the drug are the same as previously described.

(ii) Write down an equation for c and t in terms of k, γ and A.

(iii) Sketch a graph of c against t, indicating the times T and $2T$ on your graph.

(iv) Write down an equation for the concentration of drug in the GI tract against time.

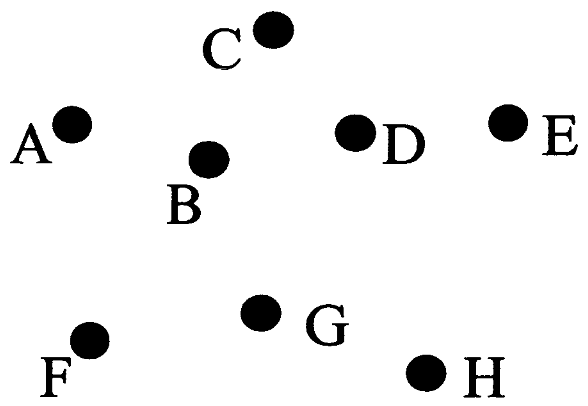
4 A university department organises optional courses for its final year undergraduates. Each course requires 3 one hour lectures per week. Devise an integer programming problem whose solution could be used to produce a timetable that minimises the total number of lecture rooms required, given the courses that students have chosen. Any assumptions you make should be clearly specified.

5 (a) (i) What is the "convex hull" in the context of a travelling salesman problem?

(ii) How and under what circumstances can the convex hull be used to evaluate a feasible solution to a travelling salesman problem?

(b) A builder needs to visit a number of potential clients in different towns labelled A, B, ... H, to provide them with quotes. She wants to minimise the distance that she travels, starting and finishing at her home town of H and visiting each other town once. The table below gives the shortest distance between each pair of towns, and each town's location is shown in the accompanying diagram.

	A	B	C	D	E	F	G	H
A	-	11	16	18	26	16	18	26
B		-	13	11	18	17	13	19
C			-	10	18	22	19	23
D				-	10	20	15	17
E					-	27	19	18
F						-	12	24
G							-	10
H								-



(i) State the nearest neighbour algorithm and apply it to this problem using the information in the table, showing each step in your working.

(ii) Show that the route obtained using the nearest neighbour algorithm is less than 10% longer than the optimal solution.