UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M310: Operational Research and Mathematical Modelling

COURSE CODE	:	**MATHM310**
UNIT VALUE	:	**0.50**
DATE	:	**15-MAY-03**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

03-C0960-3-40

TURN OVER

*All questions may be attempted but only marks obtained in the best four solutions will count. The use of an electronic calculator is **not** permitted in this examination*

1 (a) Suppose $f: R^2 \to R^1$ is a differentiable function with continuous second partial derivatives. Write down Newton's algorithm for trying to find a solution to the equation $\nabla f = 0$.

(b) Give an example of a linear programming problem with an unbounded feasible region for which the corresponding integer programming problem has no feasible solutions.

(c) The function $f: R^2 \to R^1$ is given by

$$f(x, y) = (x-1)^2 + (y-1)^2$$

(i) Show that f is a convex function.

(ii) Find where f achieves its maximum and minimum values within the set S given by

$$S = \{(x, y)^T : 0 \le x \le 1, 0 \le y \le 1\}$$

2 Five books are to be awarded to the five best final year students. The five students have expressed their individual preferences for the books by giving a whole number score for each book in the range 0 to 100. A score of 0 indicates an option that a student regards as worthless. Other scores represent acceptable options, increasing according to the students' preferences.

(i) Frame an integer programming problem whose solution could be used to decide the allocation of books to students that gives the maximum total preference score, but which might result in some students receiving books that they regard as worthless.

(ii) Frame an integer programming problem whose solution of assigning the books to students would maximise the number of students who receive acceptable books.

3 (a) A homogeneous Markov Chain has n states $S_1, S_2 .. S_n$. If $p_{ij}(m)$ denotes the probability of moving from state S_i to S_j over m stages of the chain, show that for any r such that $0 < r < m$,

$$p_{ij}(m) = \sum_{k=1}^{n} p_{ik}(r) p_{kj}(m-r)$$

(b) Amy and Beth play a game in which they take turns to toss a biased coin. If one player throws a head she gains one point and has the option of throwing again or passing the coin to the other player. If she throws a tail, all the points amassed in that turn are lost and the coin passes to the other player. The first player to amass two points wins. If Amy throws a head she will always choose to toss again whereas Beth will always keep the one point and pass the coin.

If the probability of tossing a head is p, formulate the game as a Markov Chain by defining appropriate states and writing down the associated transition matrix.

4 (a) Consider an M/M/1 queuing system with a finite capacity N for the queue and any customer being served. If $P_n(t)$ denotes the probability of there being n in the system at time t, the differential equations for the system are:

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t)$$
$$P_n'(t) = \lambda P_{n-1}(t) - (\lambda + \mu)P_n(t) + \mu P_{n+1}(t) \qquad 0<n<N$$
$$P_N'(t) = \lambda P_{N-1}(t) - \mu P_N(t)$$

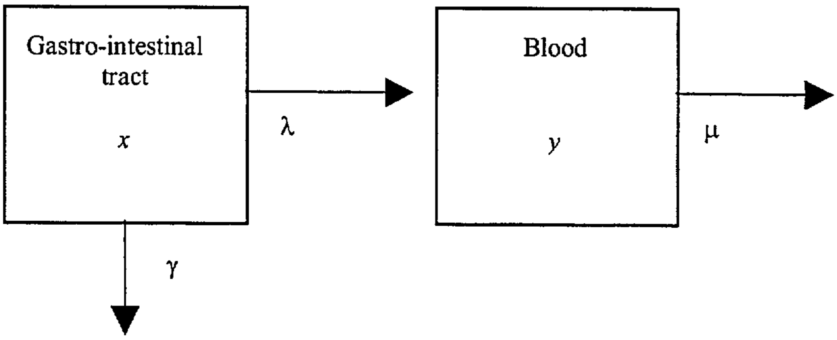
If $\rho = \lambda/\mu$, find the stationary distributions for the number in the system when $\rho = 1$ and when $\rho \neq 1$.

(b) A company produces a range of wooden boxes using the same three stage process. Anne constructs the box. Bert then cleans it before Colin paints it. These processes, which can each take a different amount of time depending on the type of box being produced, are labelled A, B and C respectively.

Each day, the company has to schedule the production of N boxes, indexed $\{1,\ldots,N\}$. To produce box i, processes A, B and C take A_i, B_i and C_i minutes respectively. Bert and Colin process the boxes in the same order as Anne. The company wants to schedule the production of boxes in the order that takes the least total time.

Explaining your reasoning, find an expression that provides a lower bound for the time taken to complete the production of the N boxes that could be useful in evaluating possible solutions to this scheduling problem.

5 A drug used to treat a blood disorder is administered by an oral dose of D grams. Once in the gastro-intestinal tract, the drug is transferred to the blood in a first order process with coefficient λ and eliminated via another route in a first order process with coefficient γ. The drug is eliminated from the blood in a first order process with transfer coefficient μ. The amounts of drug in the gastro-intestinal tract and the blood stream are labelled x and y respectively.



(i) Write down the differential equations that govern the amount of drug in the gastro-intestinal tract and in the blood stream.

(ii) Show that if a single dose of D grams is given at time t=0 and there are no further doses, the amount of drug in the blood stream is given by

$$y(t) = \frac{D\lambda}{\mu - \gamma - \lambda}\left[e^{-(\gamma+\lambda)t} - e^{-\mu t}\right]$$

In practice, treatment is delivered as a series of oral doses, each of D grams, separated by a time period τ. The drug concerned is a mild irritant and the patient's doctor wants to ensure that the amount of drug in the gastro-intestinal tract, x, does not exceed A grams.

(iii) Find an expression for the maximum dose D that can be used if this is not to occur.