UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics B46: Mathematics and Statistics for Computer Scientists

COURSE CODE	:	**MATHB046**
UNIT VALUE	:	**0.50**
DATE	:	**19-MAY-05**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

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TURN OVER

There are two sections. Full marks may be obtained by answering five questions, but no more than three questions from any one section will count.

Statistical tables are provided

The use of an electronic calculator **is** *permitted in this examination.*

Section A: Use a separate answer book for this section

1. A telephone helpline receives on average 45 calls a day. To boost its use, its phone number is advertised in the London Underground during a period of one month. A year later, the number of phone calls to the helpline per day is recorded on 10 consecutive working days. The following data are obtained:

$$46, 58, 71, 48, 51, 57, 39, 53, 62, 51.$$

   (a) Find the median, the upper and lower quartiles, and the interquartile range of these data.

   (b) Calculate the sample mean, the sample variance and the coefficient of variation.

   (c) In order to further investigate whether the advertising has led to an increased use of the helpline, the above sample is enlarged by recording the number of phone calls to the helpline per day on a further 25 working days. For the total sample of 35 working days, a sample mean of 50.88 phone calls per day and a sample variance of 78.32 are found. Perform an appropriate statistical hypothesis test, at a significance level of 1%, to assess whether the true mean number of phone calls to the helpline per day is now higher than it was before the advertising campaign. Please state any assumptions that you make and formulate your conclusion clearly.

2. An experiment consists of randomly rearranging the 9 letters of the word

TARANTULA

into a sequence of 9 letters, where all possible orders of these 9 letters are equally likely. Find the probability of each of the following events:

(a) the first three letters include no A's;

(b) the first three letters or the last three letters (or both) include no A's;

(c) the fourth letter is the first A;

(d) the first letter and the last letter are the same;

(e) the word 'TARANTULA' is obtained;

(f) the sequence contains the word 'RAT'.

3. Two factories, A and B, produce apparently identical diskettes. However, each diskette made in factory A has a probability of 0.02 of being defective (independently of all other diskettes), while the corresponding figure for diskettes made in factory B is 0.03.

(a) Let X be the number of diskettes made in factory A, up to and including the first defective one. Name the distribution of X and state its mean. Find $P(X = 5)$ and $P(X > 50)$.

(b) Approximately calculate the probability that amongst the first 1000 diskettes made in factory A, there are more than 25 defective diskettes.

Of the diskettes for sale in a particular shop, 60% come from factory A and 40% come from factory B. The diskettes are sold in boxes of 10, where all diskettes in the same box are from the same factory. All boxes of diskettes look identical. A student buys a randomly chosen box of diskettes from this shop.

(c) The student randomly takes a diskette from the box. Calculate the probability that this diskette is defective.

(d) Suppose that the box is found to contain exactly one defective diskette. Given this information, which factory is most likely to have produced this box of diskettes, and with what probability?

4. The time X (in hours) needed to repair a computer is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{9x^4} & \text{if } x \geqslant 1/3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Calculate $E(X)$ and $\mathrm{Var}(X)$.

(b) The cost (in pounds) of having the computer repaired by engineer A is given by $C_A = 30 + 50X$. Find the mean and the standard deviation of C_A.

(c) The cost (in pounds) of having the computer repaired by engineer B is given by

$$C_B = \begin{cases} 50 & \text{if } X \leqslant 1/2 \\ 80 & \text{if } X > 1/2. \end{cases}$$

Find the mean and standard deviation of C_B.

(d) Suppose that 50 computers need to be repaired by a single engineer, and that their repair times $X_1, X_2, \ldots, X_{50}$ are independent random variables, each with the same distribution as X above. Approximately calculate the probability that the engineer needs more than 24 hours to repair all 50 computers.

Section B: Use a separate answer book for this section

5. (a) Prove, from the definition of the derivative, that the derivative of the function $y = x^2 + \sqrt{x}$ equals $2x + 1/(2x^{1/2})$ for $x > 0$.
   (b) Prove that if f and g are differentiable functions, then $(fg)' = f'g + g'f$.
   (c) Find the absolute value and the argument of each of the following complex numbers: $-3 + 4i$; e^i; $(1+i)^{-1}$; $\cos(i)$.

6. (a) Find the following limits:
   (i)
   $$\lim_{x\to 0} \frac{2 + e^{3x} - 3e^x}{x\sin(5x)};$$
   (ii)
   $$\lim_{x\to \pi} \frac{\cos x + 1}{\sin^2 x};$$
   (iii)
   $$\lim_{x\to 1} \frac{\int_1^x \ln t dt}{(x-1)^2}.$$
   (b) Find the Taylor series expansion of $f(x) = \frac{1}{(x+2)^2}$ at the point $x = 0$.

7. (a) Find all stationary points of the function $f(x, y) = e^{x(x+y)}(x - y)$ and classify them as local maxima, local minima or saddle points.
   (b) Find the volume of the 3-dimensional region which consists of the points satisfying the following inequalities: $0 \leq x \leq \pi$, $0 \leq y \leq x$, $0 \leq z \leq \sin(\pi y/x)\sin x$.

8. Let $f(x) = (1 - x^2)e^x$.
   (i) Find extremal points (i.e. points of local maximum or local minimum) of f.
   (ii) Draw the graph of the function f.
   (iii) Find the area of the region located above the x-axis, but below the graph of f.