UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Mathematics B46: Mathematics and Statistics for Computer Scientists

COURSE CODE	:	**MATHB046**
UNIT VALUE	:	**0.50**
DATE	:	**10-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0902-3-90

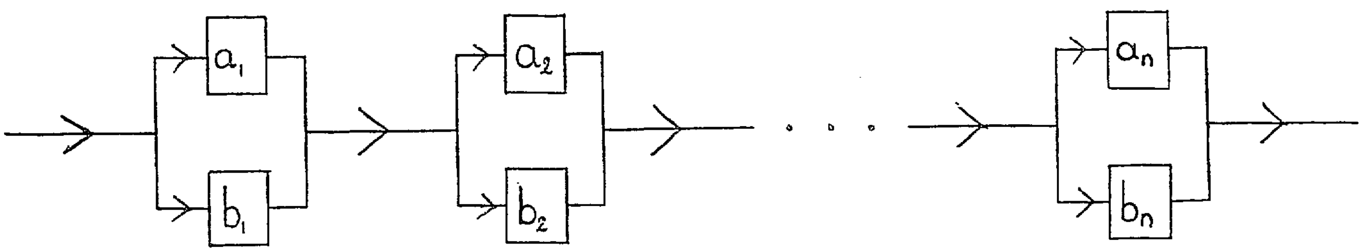
TURN OVER

Answer **at least two questions** *from Section A and* **at least two questions** *from Section B, and* **one further question** *from either section.*
Statistical tables are provided.

The use of an electronic calculator **is** *permitted in this examination.*

SECTION A: Use a separate answer book for this section

1. A circuit is made up of n 'blocks' wired in series, where each 'block' consists of 2 components wired in parallel:



Assume that the $2n$ components $(a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n)$ have a failure probability of 5% each, independently of each other. A block works if at least one of the two components in that block works. The circuit works if all blocks work.

(a) Find the probability that the first block works.

(b) Find the probability that the circuit works.

(c) Find the maximum number of blocks allowed (i.e. the maximum value of n) if the probability that the circuit works should be at least 97%.

(d) Find the probability that exactly 2 blocks fail in a circuit of length $n = 20$ blocks.

2. In a bid to increase government funding for its super-computer, a university claims that on average it has 1000 login requests per working day. A government verification officer suspects that this claim may be exaggerated. In order to verify the university's claim, he records the number of login requests on 10 working days, obtaining the following data:

$$872, \ 985, \ 1011, \ 668, \ 1132, \ 1009, \ 998, \ 1221, \ 827, \ 915.$$

It is assumed that these data can be regarded as an independent random sample from a distribution that is approximately normal.

(a) Calculate the sample mean and standard deviation of the above data.

(b) Find the median of the above data.

(c) Perform an appropriate statistical hypothesis test, at a significance level of 5% to evaluate whether the above data provide significant evidence that the true mean number of login requests per working day is less than 1000. State your conclusion clearly.

(d) Construct a 95% confidence interval for the true mean number of login requests per working day.

3. *(Recall that a discrete random variable X has a Poisson distribution with mean μ if $P(X = k) = \dfrac{e^{-\mu}\mu^k}{k!}$ for $k = 0, 1, 2, \ldots$. In that case $\mathrm{Var}(X) = \mu$ as well.)*

A university department has 2 secretaries who do all the word processing required by the department. The number of misprints on a randomly sampled page typed by secretary A has a Poisson distribution with mean 0.4 (independently of the number of misprints on any other page). The number of misprints on a page typed by secretary B has a Poisson distribution with mean 0.9 (and is again independent of the number of misprints on any other page). Of the typing required by the department, 35% is done by secretary A and 65% is done by secretary B.

(a) Calculate the probability that a randomly sampled page typed by secretary B contains at least 1 misprint.

(b) Calculate the overall proportion of pages produced by the department that contain no misprints.

(c) Suppose that a randomly sampled page produced by the department is found to contain no misprints. Calculate the (conditional) probability that this page was typed by secretary A.

(d) A book typed by secretary B consists of 120 pages. The number of misprints on each page is counted and the average number of misprints per page of the book is calculated; denote this average by $\bar{X}$. Name the approximate distribution of $\bar{X}$ and approximately calculate $P(\bar{X} > 1)$.

4. *(Recall that an exponentially distributed random variable X with mean $E(X) = 1/\lambda$ has probability density function $f(x) = \lambda e^{-\lambda x}$ for $x > 0$, and cumulative distribution function $F(x) = 1 - e^{-\lambda x}$ for $x > 0$.)*

A machine is made up of two components. The time T_1 until the first failure of component 1 is exponentially distributed with a mean of 8000 hours; the time T_2 until the first failure of component 2 is exponentially distributed with a mean of 5000 hours. Assume that T_1 and T_2 are independent of each other. The machine functions if and only if both components work.

(a) Calculate the probability that after 6000 hours, component 1 has not yet failed.

(b) Calculate the probability that after 6000 hours, the machine has not yet failed.

(c) Denote by T the time until the first failure of the machine. Find the cumulative distribution function of T. Name the distribution of T and state its mean.

(d) Given that component 1 has not yet failed after 6000 hours, find the (conditional) probability that this component does fail within the first 9000 hours.

Section B: Use a separate answer book for this section

5. (a) Prove, from the definition of the derivative, that the derivative of the function $y = x^4 + 5x$ at $x = -1$ is 1.

   (b) Define the function $\arctan(x)$, stating for which values of x your function is defined. Show that the derivative of $\arctan(x)$ is $1/(1+x^2)$.

   (c) Find all solutions of the differential equation $y'' + 4y' - 5y = 0$.

6. (a) Let $f(x, y, z) = \log((x^2 + e^y)\tan z) + \cos(e^{x^{-1}+z}) + \frac{1}{\sqrt{x+y+z^2}}$. Compute all the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$.

   (b) Solve the differential equation $y' = y^2 \log x$ given that $y = 2$ when $x = 1$.

7. (i) Find all stationary points of the function $y = (1 - \log x)x$ and classify them as local maxima, local minima or points of inflexion.

   (ii) Sketch the graph of y.

   (iii) Find the area of the region which lies below the graph but above the x-axis.

8. (a) Among all 7-digit numbers (i.e. numbers between 1000000 and 9999999 inclusive):

      (i) How many have all 7 digits distinct?

      (ii) How many contain exactly two 3 but no more than one 0?

      (iii) How many have precisely 6 distinct digits, all of them non-zero?

   (b)(i) State the binomial theorem concerning the expansion of $(x+y)^n$ where x and y are real numbers and n is a non-negative integer.

   (b)(ii) Hence show that for any positive integer n the sum

$$\binom{n}{0} - 4\binom{n}{1} + 4^2\binom{n}{2} - \cdots \pm 4^n\binom{n}{n}$$

is divisible by 3.

END OF PAPER