UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:–

B.Sc. *M.Sci.*

Mathematics C398: Mathematics in Biology 2

COURSE CODE : **MATHC398**

UNIT VALUE : **0.50**

DATE : **19–MAY–04**

TIME : **10.00**

TIME ALLOWED : **2 Hours**

04–C0940–3–40

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Consider the selection model with n alleles $A_1, A_2, \ldots, A_n$ in a large, randomly mating, diploid population:

   a) How do the frequencies $p_1, p_2, \ldots, p_n$ evolve from one generation to the next? Explain the relevant parameters (fitness of a genotype, etc.)

   b) State, without proof, the fundamental theorem of natural selection.

   c) Show how this theorem implies that each orbit approaches the set of fixed points.

   d) If $n = 3$, what can be said about the number of fixed points?

   e) Explain how the asymptotically stable fixed points of the selection map can be found and characterized from the mean fitness function.

   f) Analyze the example with two alleles A, a where the fitnesses of genotypes AA, Aa, aa are given by $0.8, 1.0, 0.0$, respectively.

2. In a large, randomly mating population consider a gene locus on the X-chromosome that allows for two alleles A and a. Suppose in males (that carry only one such allele) a is lethal, so that fitnesses of A and a are 1 and 0, respectively. In females, fitnesses of genotypes AA, Aa, aa are denoted as w_{AA}, w_{Aa}, w_{aa}.

   a) Derive the equations for the change of allele frequencies in males and females.

   b) What happens with allele frequencies in males?

   c) What are the equilibrium allele frequencies, i.e., the fixed points of this map?

   d) For what fitness values does a polymorphic equilibrium exist? Why does the result not depend on w_{aa}?

3. Explain the process of recombination. Consider alleles $A_1, \ldots, A_n$ at one locus and alleles $B_1, \ldots, B_m$ at another locus, and let x_{ij} be the frequency of gametes A_iB_j.

   a) If the probability for recombination between these two loci is r derive the frequencies x'_{ij} in the next generation.

   b) What values can r take?

   c) Show that x_{ij} converges over generations, and determine the limit. Does the limit depend on the initial condition?

   d) Describe the linkage equilibrium manifold and sketch it in the case $n = m = 2$.

4. Consider a symmetric, two-player game with n strategies labelled $1, 2, \ldots, n$ and payoff matrix $A = (a_{ij})$.

   a) Define the following terms associated with this game: Nash equilibrium, strict equilibrium, evolutionarily stable strategy (ESS).

   b) What is the logical relation between these equilibrium concepts?

   c) Write down the replicator dynamics and provide some intuition for it.

   d) Show that a Nash equilibrium is an equilibrium for this dynamics.

   e) Show that an ESS is an asymptotically stable equilibrium for this dynamics.

   f) Compute all dynamic equilibria, Nash equilibria and ESS for the following game:

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

5. Analyze the following variant of Dawkins' *battle of the sexes* game, with males being either philandering or faithful, and females being either fast or coy. Suppose, that the gain for a successfully raised offspring is 15 to each parent, the cost of parental investment is 20 (which is equally shared by both parents if the male is faithful, and otherwise borne by the female alone) and the cost for an engagement period is 3 and 1 to (faithful) males and (coy) females, respectively.

   a) Set up the payoff matrices for males and females.

   b) Determine all Nash equilibria.

   c) Write down the replicator dynamics for this game.

   d) Sketch the corresponding vector field.

   e) Show that there is a constant of motion.

   f) What does this constant of motion imply for the solutions?