UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. B.Sc. B.Sc.(Econ)M.Sci.

Mathematics B51B: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB51B**
UNIT VALUE	:	**0.50**
DATE	:	**12-MAY-06**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

06-C1002-3-190

TURN OVER

Answer ALL questions from Section A.

All questions from Section B may be attempted, but only marks obtained on the best **two** *solutions from Section B will count.*

The use of an electronic calculator is **not** *permitted in this examination.*

Section A

1. (a) Maximize $f(x,y) = 2x^{1/3}y^{2/3}$ subject to $4x + y = 12$.

   (b) Maximize and minimize $g(x,y) = x^2 + 2y^2$ subject to $x \geqslant -2$, $y \leqslant 3$ and $y \geqslant x - 2$.

   (c) Maximize $h(x,y) = x^2 + 3y^2$ subject to $x^2 + y^2 \leqslant 9$.

2. (a) Find an invertible matrix P such that $P^{-1}AP$ is diagonal, where A is the matrix given below
$$A = \begin{bmatrix} 5 & -4 \\ 2 & -1 \end{bmatrix}.$$

   (b) Solve the following system of simultaneous difference equations subject to the initial conditions $x_0 = 2$ and $y_0 = 0$
$$\begin{aligned} x_{n+1} &= 3x_n + y_n, \\ y_{n+1} &= x_n + 3y_n. \end{aligned}$$

3. (a) Solve the differential equation
$$x^3 + \frac{\mathrm{d}y}{\mathrm{d}x}(y+1)^2 = 0,$$
subject to the initial condition $y = -1$ at $x = 0$.

   (b) Solve the differential equation
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x+y}{4-3x-3y},$$
subject to the initial condition $y = 1$ at $x = 0$.

Section B

4. (a) Solve the difference equation

$$x_{n+2} - 4x_{n+1} + 3x_n = -6,$$

subject to the initial conditions $x_0 = 3$ and $x_1 = 8$.

(b) Solve the difference equation

$$x_{n+2} - 4x_{n+1} + 4x_n = 2^n,$$

subject to the initial conditions $x_0 = 3$ and $x_1 = \frac{33}{4}$.

5. (a) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x} = y^3(1 + \ln x),$$

corresponding to the initial condition $y = 1$ at $x = 1$.

(b) Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 5\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 20\cosh x,$$

corresponding to the initial conditions $y = 1$ at $x = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{22}{3}$ at $x = 0$.

6. (a) Evaluate the following integral, where R is the region of the plane satisfying $0 \leqslant x \leqslant 3$ and $0 \leqslant y \leqslant 4$

$$\iint_R xy \, \mathrm{d}A.$$

(b) Let $k \geqslant 1$ be an integer. Evaluate the following integral, where S is the entire plane

$$\iint_S (x^2 + y^2)^k e^{-(x^2+y^2)^{k+1}} \, \mathrm{d}A.$$

(c) Using the change of variables $u = x + y$ and $v = y/(x + y)$, or otherwise, evaluate the following integral, where T is the region given by $0 \leqslant y \leqslant 1 - x$ and $0 \leqslant x \leqslant 1$

$$\iint_T e^{y/(x+y)} \, \mathrm{d}A.$$

7. (a) Find the Maclaurin series expansion of $f(x) = \frac{1}{1+x}$.

   (b) Evaluate the following integral. Your final answer should not contain the gamma function.

$$\int_0^1 \sqrt{\ln\left(\frac{1}{x}\right)}\,dx.$$

   (c) Express the complex number $(1+i)^{1+i}$ in Cartesian form.