UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *B.Sc.(Econ)M.Sci.*

Mathematics B51B: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB51B**
UNIT VALUE	:	**0.50**
DATE	:	**16-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

05-C0917-3-240

TURN OVER

Answer ALL questions from Section A.
All questions from Section B may be attempted, but only marks obtained on the best **two** *solutions from Section B will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

Section A

1. (a) Evaluate the integral
$$\iint_R \sqrt{x^2+y^2}\,\mathrm{d}A,$$
where R is the region of the plane satisfying $1 \leqslant x^2+y^2 \leqslant 9$.

   (b) Sketch the region S of the positive quadrant bounded by $x^2-y^2=1$, $x^2-y^2=16$, $xy=2$ and $xy=4$.
   By using the change of variable $u=x^2-y^2$ and $v=2xy$ or otherwise evaluate the integral
$$\iint_S (x^2+y^2)\,\mathrm{d}A,$$

2. (a) Solve the differential equation
$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2+2xy+y^2,$$
subject to the initial condition $y=0$ at $x=0$.

   (b) Solve the differential equation
$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = \frac{\ln x}{x},$$
subject to the initial condition $y=0$ at $x=1$.

3. (a) Find an invertible matrix P such that $P^{-1}AP$ is diagonal, where A is the matrix given below.
$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}.$$

   (b) Solve the following system of simultaneous differential equations subject to the initial conditions $x_1(0)=1$ and $x_2(0)=0$
$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}t} &= 4x_1-2x_2, \\ \frac{\mathrm{d}x_2}{\mathrm{d}t} &= x_1+x_2. \end{aligned}$$

Section B

4. (a) Solve the difference equation

$$x_{n+2} - 2x_{n+1} + x_n = 8,$$

subject to the initial conditions $x_0 = 1$ and $x_1 = 7$.

(b) Solve the difference equation

$$x_{n+2} + x_n = 2n + 4,$$

subject to the initial conditions $x_0 = 6$ and $x_1 = 9$.

5. (a) Find the minimum of $x_1^2 + x_2 + x_3^2 + x_4^2$ subject to the constraints

$$x_1 + x_2 + x_3 = 3 \quad \text{and} \quad x_2 + x_4 = -1.$$

(b) Find the minimum and maximum values of $x^2 + 3y^2$ subject to the constraints $x \geqslant -1$, $y \geqslant -5$ and $y \leqslant 4 - x^2$.

6. (a) Find the solution of the differential equation

$$2xy\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y^2,$$

corresponding to the initial condition $y = 0$ at $x = 1$.

(b) Find the solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = 2\cos x \sin x + 8\cos 2x,$$

corresponding to the initial conditions $y = 4$ at $x = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = 0$.

7. (a) Find the first four terms of the Maclaurin series expansion of $f(x) = \ln(a + x)$, where $a > 0$ is a constant.

(b) Evaluate the following integral. Your final answer should not contain gamma or beta functions. (Any properties of the gamma and beta functions that you use should be clearly stated.)

$$\int_0^{\pi/2} \cos^5 x \sin^6 x \, \mathrm{d}x.$$

(c) Let A_n and B_n be given by the binomial coefficients $A_n = \binom{4n}{2n}$ and $B_n = \binom{6n}{n}$. Given that $5^5 > 36 \times 3^4$ use Stirling's formula to find

$$\lim_{n\to\infty} \frac{A_n}{B_n}.$$

MATHB51B END OF PAPER