UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics A3: Mathematics For Physical Science

COURSE CODE	:	**MATHA003**
UNIT VALUE	:	**0.50**
DATE	:	**14-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

04-C0903-3-40

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) If $z_1 = 2 + i$, $z_2 = -2 + 4i$ and

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2},$$

find z in the form $a + bi$. Also, find the modulus and the argument of z.

(b) State de Moivre's theorem, and then use it to find the double-angle formulae ($\sin(2\theta) = \cdots, \cos(2\theta) = \cdots$).

(c) Find the cartesian form of the equation

$$(z - \bar{z})^2 = -8(z + \bar{z}),$$

where $z = x + iy$. Sketch the graph of the equation.

2. (a) Consider a triangle ABC. Let M and N be respectively the midpoints of sides AB and AC. Using vectors, prove that MN is parallel to BC and has half its length.

(b) Consider a quadrilateral $ABCD$, whose vertices are $A = (\alpha, 6, 3\alpha+1)$, $B = (1, 1, -1)$, $C = (0, 6, 1)$ and $D = (-7, 11, -15)$, where α is a non-zero real number. Check that A, B, C, D all lie on the same plane, and give the equation of this plane in cartesian form. Use vectors to find for which value of α the quadrilateral $ABCD$ is a rectangle.

3. (a) Find the equation of the plane normal to $\underline{n} = (1, -2, -1)$ passing through $(2, 4, -3)$.

(b) Find a parametric equation for the line passing through $(1, -1, 2)$ parallel to the vector $\underline{b} = (1, 2, 1)$.

(c) Find the point where the line in (b) meets the plane in (a).

(d) Find the cosine of the angle between $\underline{n}$ and $\underline{b}$.

4. (a) Differentiate the following with respect to x:

$$y = \cos^{-1}(x^4), \qquad y = \tan^{-1}(2\sqrt{x}), \qquad y = \sin^{-1}\left(\frac{1}{x}\right).$$

(b) Calculate the following integrals,

$$\int_0^1 \frac{x}{1+x^4}\,dx, \qquad \int_0^{\pi/2} \frac{1}{3+2\sin\theta}\,d\theta, \qquad \int_0^{\pi/4} \cos^4\theta\,d\theta.$$

5. Find Maclaurin series up to and including x^5 for e^x, $\cos x$ and $\sin x$. Use these series to verify that

$$e^{i\theta} = \cos\theta + i\sin\theta$$

as far as terms in θ^5.

6. Use Gaussian elimination to find the solution of the simultaneous equations

$$\begin{aligned} x_1 - x_2 - 4x_3 &= 1, \\ 2x_1 + 5x_2 - x_3 &= 2, \\ 3x_1 + 2x_2 - 3x_3 &= -1. \end{aligned}$$

Explain briefly how different real number values of c determine the type of solution of the simultaneous equations

$$\begin{aligned} x_1 - 3x_2 + x_3 &= c, \\ 2x_1 + 4x_2 - 2x_3 &= 3, \\ 2x_1 - 6x_2 + 2x_3 &= 6. \end{aligned}$$

7. An object is thrown horizontally from the top of a building (100 metres high) with initial speed $v = 20$ (metres/second). Determine the path of the object. When and where does it hit the ground? (The acceleration g due to gravity is taken to be $9.81 m/s^2$).

END OF PAPER