UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Eng. *B.Sc.* *M.Sci.*

Mathematics A3: Mathematics For Physical Science

COURSE CODE	:	**MATHA003**
UNIT VALUE	:	**0.50**
DATE	:	**24-MAY-02**
TIME	:	**10.00**
TIME ALLOWED	:	**2 hours**

02-C0894-3-50

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) State de Moivre's theorem and use it to derive expressions for $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$.

   (b) Find the solutions of
$$z^4 = \frac{1-i}{1+i},$$
in the form $z = r(\cos\theta + i\sin\theta)$ and draw them on an Argand diagram.

2. (a) Find the vector and cartesian equations of the plane which passes through the points $(1,2,-1)$, $(2,3,1)$ and $(3,-1,2)$.

   (b) Find the point of intersection of the two lines
$$\begin{aligned} \mathbf{r}_1 &= (1,0,1) + t_1(1,2,3) \\ \mathbf{r}_2 &= (1,-2,1) + t_2(\frac{1}{3},1,1) \end{aligned}$$

3. (a) Write down the equation of the circle C in the xy-plane, which has its centre at the point $(3,-1)$ and radius 2.

   (b) Find the equations of the lines which pass through the point $(0,1)$ and are tangent to the circle C.

   (c) Find the gradient of the line passing through $(0,1)$ and the centre of the circle C.

4. (a) Differentiate the following with respect to x:

$$y = \sin^{-1}(\frac{1}{x}), \qquad y = \tan^{-1}(3x^2).$$

(b) Find the following integrals:

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{3-x^2}} \mathrm{d}x, \qquad \int_0^{\frac{\pi}{2}} \frac{1}{2+\cos\theta} \mathrm{d}\theta, \qquad \int \frac{2x}{x^2-16} \mathrm{d}x.$$

5. (a) Derive the MacLaurin series for $\sin x$ and $\ln(1+x)$ up to and including terms of order x^5.

(b) Using these and assuming the geometric series

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}, \quad |r| < 1,$$

find the MacLaurin series for
(i)

$$f(x) = \frac{\sin x}{1-x^2},$$

(ii)

$$f(x) = \frac{\ln(1+x^2)}{1+x},$$

up to and including terms of order x^4.

6. Solve each of the following systems of equations using Gaussian or Gauss-Jordon elimination, giving all solutions or showing there is no solution.

(a)

$$\begin{aligned} x + y - z &= 7, \\ 4x + 5y + 5z &= 4, \\ 6x + 7y + 3z &= 18. \end{aligned}$$

(b)

$$\begin{aligned} x - y - 2z &= -3, \\ x \qquad + 2z &= 3, \\ 2x - 3y - 7z &= -11. \end{aligned}$$

(c)

$$\begin{aligned} 3x + y + 3z &= 1, \\ x + y + 2z &= 1, \\ 4x + 2y + 5z &= 3. \end{aligned}$$

7. (a) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{4}{1+x}y = 3, \quad y(0) = 2.$$

(b) An infectious disease is introduced into a population. Suppose $P(t)$ is the proportion of people exposed to the disease within t years of its introduction. If

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{(2 - P(t))}{4}$$

and $P(0) = 0$, after how many years will 80% of the population be infected? (Assume $\ln \frac{5}{3} \approx 0.5$.)