UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *M.Sci.*

Mathematics A2: Differential And Integral Calculus

COURSE CODE	:	**MATHA002**
UNIT VALUE	:	**0.50**
DATE	:	**06-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Differentiate the following with respect to x:

   (a) $\cos^2(x/2+4)$,

   (b) $\frac{x^2+5}{3x-4}$,

   (c) $x^3\ln(3x^{4/3}+2x^{1/2})$,

   (d) $\exp(2\tan(2\sqrt{x}))$,

   (e) $\sin(\sin(\sin(x)))$.

2. Find the minimal possible surface area of a cylindrical barrel that is open at one end and has a capacity of $1000\pi m^3$.

3. In the first hour a culture of E.coli. bacteria grows in number from 2000 to 8000. Assuming that a simple exponential growth model applies, find a formula for the number of bacteria in the culture at time t hours. What is the number after 3 hours?

4. Draw a graph of the function $y=x^2-4x$ and find the total area which lies between the curve $y=x^2-4x$, the x axis and the lines $x=-2$ and $x=3$.

5. Compute the following integrals:

(a)
$$\int_0^1 \frac{1}{\sqrt{2-x}}dx,$$

(b)
$$\int_1^2 x^3 \ln x dx,$$

(c)
$$\int_1^3 \frac{x}{x^2-3x-4}dx,$$

(d)
$$\int_0^{\pi/3} \frac{\tan x}{\cos^2 x}dx.$$

6. The distance D travelled by a moving object from time $t = a$ to time $t = b$ is given by

$$D = \int_a^b v(t)dt,$$

where $v(t)$ is the velocity at time t.

Use the trapezium rule to estimate the distance travelled by any object between time $t = 0$ and $t = 6$, given the following values of $v(t)$.

t(secs)	0	1	2	3	4	5	6
$v(t)$(metres/sec)	3	4	3	5	7	8	10

7. Solve the following initial-value problems:

(a)
$$(1+x^2)\frac{dy}{dx} + 2xy^2 = 0, \quad y(0) = 2.$$

(b)
$$y'' + 2y' + y = 2e^{-x}, \quad y(0) = 1, \quad y'(0) = 0.$$

END OF PAPER