UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *M.Sci.*

Mathematics A2: Differential And Integral Calculus

COURSE CODE	:	**MATHA002**
UNIT VALUE	:	**0.50**
DATE	:	**11-MAY-06**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Differentiate the following with respect to x:

   (a) $x^2 \sin\left(-\frac{1}{x}\right)$, (b) $\exp(x)\ln(x^2)$, (c) $\exp(\cos^2(x))$,

   (d) $\sin\left(\frac{\exp(x)}{1+x^2}\right)$, (e) $\ln\left(x^{\cos(x)}\right)$.

   Express your answers in their simplest forms.

2. (a) State the definition of the derivative of a function $f(x)$. From this definition find the derivatives of the following functions

   $$(1)\ f(x) = \frac{1}{x}, \qquad (2)\ f(x) = \frac{1}{x^2}.$$

   (b) State the quotient rule for differentiating a quotient. Prove the quotient rule using the product rule, the chain rule and one of the results obtained in part (a).

3. Consider a rectangular metal sheet with dimensions 12 metres by 6 metres. Squares of size x metres by x metres are cut off the corners of the rectangular sheet (and thrown away) so that the rest of the sheet can be folded into an open top box.

   (a) By means of a sketch show the construction described above.

   (b) Find the expression for the volume of the final open top box as a function of x. Sketch a graph of this function.

   (c) Determine the value of x which gives the maximum box volume, and determine the volume in this case.

4. An insect population has 10^3 members in week 0 and by week 2 has 10^5 members, where measurements of the insect population are taken at the same time and day of each week. Assuming a simple exponential growth model, determine the growth rate of the population and determine the population size at the following times:

   (a) week 1, (b) week 3, (c) week -1, (d) week -4.

   Is the exponential growth model likely to be valid for week -4? Determine how long it takes for the population to double.

5. Compute the following integrals:

   $$(a)\ \int_0^{\pi/4} \cos(2x)\sin^3(2x)\,dx, \qquad (b)\ \int_1^2 x(\ln x)^2\,dx,$$

   $$(c)\ \int_0^{\sqrt{3/2}} \frac{1}{\sqrt{3-2x^2}}\,dx, \qquad (d)\ \int_3^4 \frac{3x}{x^2-3x+2}\,dx.$$

6. The work done, W, by a force, F, in moving a body through a distance from $x = a$ metres to $x = b$ metres is given by

   $$W = \int_a^b F\,dx \quad \text{Joules.}$$

   Suppose the following values for F are given:

x (metres)	0	1	2	3	4	5	6	7	8
F (Newtons)	0	0.4	0.7	0.9	1.3	1.6	2.1	2.8	3.8

   Use the trapezium method with 4 and 8 intervals of equal length to give two estimates of the work done by the force in moving a body from $x = 0$ to $x = 8$ metres.

7. (a) Solve the following initial-value problem:

$$y' + y^2 = 0, \quad y(1) = 1.$$

(b) Find the general solution of

$$y'' + 4y' + 4y = \sin(2x).$$