UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *M.Sci.*

Mathematics A2: Differential And Integral Calculus

COURSE CODE	:	**MATHA002**
UNIT VALUE	:	**0.50**
DATE	:	**11-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

05-C0906-3-40

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Differentiate the following with respect to x:
   (a) $\sqrt{\sin(\sqrt{x})}$,
   (b) $\dfrac{x-2}{3x^2-2x+1}$,
   (c) $\ln(x^2)+2\ln(x+4)$,
   (d) $\exp(\sin(\cos(x^2)))$,
   (e) $\exp(x^2)\sin(x^2)\cos(x^2)$.

2. A cylindrical water tank with no lid is to be built from $150\pi cm^2$ of sheet iron. Find the radius of the base and the height required to maximise the volume of the tank.

3. A radioactive isotope has a half-life of 1000 years.
   (a) Calculate the decay constant for the isotope, and write down a formula for the proportion of a sample that remains after t years.
   (b) Use your formula to calculate how much of the sample remains after 2000 years, 3000 years and 4000 years respectively.

   For the purposes of the following question, you are given the following table of approximations to the logarithm.

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\ln x$	-2.3	-1.6	-1.2	-0.9	-0.7	-0.5	-0.36	-0.2	-0.1

   (c) Use your formula to find out how long it takes for 90% of a sample to decay.

4. Find the total area of the region bounded by $y=\frac{2}{x}$, $y=x-1$, $x=1$ and $x=4$.

5. Compute the following integrals:

   (a)
$$\int_0^1 \frac{1}{1+\sqrt{x}}dx,$$

   (b)
$$\int_1^4 x^2 \ln x dx,$$

   (c)
$$\int_1^3 \frac{5x-6}{x^2-4x-12}dx,$$

   (d)
$$\int_0^1 x^2 \exp(-x^3)dx.$$

6. Using the trapezium method with 4 intervals of equal length find the approximate value of the following integral:
$$\int_1^3 \frac{1}{x+1}dx.$$

   By means of a sketch, illustrate the areas that correspond to the above integral and the trapezium method, and thus show that the trapezium method gives an over-estimate in this case.

   Hence or otherwise find an approximation to $\ln(4)$.

7. Solve the following initial-value problems:

   (a)
$$yy' = x, \quad y(0) = 1.$$

   (b)
$$y'' + 3y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 2.$$