UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *M.Sci.*

Mathematics A2: Differential And Integral Calculus

COURSE CODE	:	**MATHA002**
UNIT VALUE	:	**0.50**
DATE	:	**19-MAY-03**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Differentiate the following with respect to x:

   (a) $(\cos(x))^{\frac{1}{2}}$,

   (b) $x^2 \ln(x)$,

   (c) $(1+5x^3)^{-\frac{2}{3}}$,

   (d) $\ln(3x^2)$,

   (e) $x^3 \exp(3x+2)$,

   (f) $\exp(x)\sin(x)$,

   (g) $\exp(\cos(x))$.

2. (a) State the definition of the derivative of a function $f(x)$. From this definition find the derivative of the function

$$f(x) = \frac{1}{x}.$$

   (b) State and prove the rule for differentiation of a quotient, using the product rule, chain rule and the result from the previous question.

   (c) Differentiate

$$\frac{e^{-x}}{x^2},$$

   using the quotient rule.

3. The five rectangular faces of an open box have a total surface area 216 square centimetres, and the box's length is twice its width.

   (a) Find the expression for the volume of the open box as a function of the width. Sketch a graph of this function and indicate the possible range for the width.

   (b) Find the width, length and height of the box which gives it maximum volume, and determine the volume in this case.

4. For the purposes of this question, you are given the following table of approximations to the logarithm.

x	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
$\ln x$	0.00	0.22	0.41	0.60	0.69	0.81	0.92	1.01	1.10

The population of India was estimated to be 0.6 billion in 1974 and 0.75 billion in 1984. Assuming a simple exponential growth model,

(a) determine the growth rate of the population,

(b) determine the expected population size in 2004,

(c) when will the population reach 1.5 billion ?

5. Compute the following integrals:

(a) $\int_0^1 x^2 \exp(-x^3) dx$,

(b) $\int_e^3 \frac{1}{x \ln x} dx$,

(c) $\int_1^3 \frac{x^4+1}{x^2} dx$,

(d) $\int_0^{\frac{\pi}{2}} \cos^3(\theta) d\theta$.

6. Using the trapezium method with 4 intervals of equal length find the approximate value of the following integral:

$$\int_1^3 \frac{1}{x^2} dx.$$

Draw a graph of the function $y = 1/x^2$ in the appropriate range and show on the same picture, the area corresponding to your estimate.

Is your estimate an under-estimate or an over-estimate ? Explain your answer by the geometrical interpretation.

7. Solve the following initial-value problems:

(a)

$$y' = 3x^2 e^{-y}, \quad y(0) = 2.$$

(b)

$$y'' + 2y' + y = x, \quad y(0) = 0, \quad y'(0) = 0.$$