# UNIVERSITY COLLEGE LONDON

*University of London*

## EXAMINATION FOR INTERNAL STUDENTS

*For the following qualifications :-*

*B.A.* *B.Eng.* *B.Sc.* *M.Sci.*

**Mathematics A2: Differential And Integral Calculus**

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|---|---|---|
| COURSE CODE | : | **MATHA002** |
| UNIT VALUE | : | **0.50** |
| DATE | : | **09-MAY-02** |
| TIME | : | **14.30** |
| TIME ALLOWED | : | **2 hours** |

02-C0893-3-70


**TURN OVER**

*All questions may be attempted but only marks obtained on the best* **five** *solutions will count.*
*The use of an electronic calculator is* **not** *permitted in this examination.*

1. Differentiate the following with respect to $x$:

   (a) $(1+x)^{-2}$,
   (b) $(\sin(x))^{\frac{1}{3}}$,
   (c) $\cos(\cos(\cos(x)))$,
   (d) $\sin(\cos(x))$,
   (e) $\exp(x)\exp(x^2)$,
   (f) $\ln(x^3)$,
   (g) $(\ln(x))^3$.

   Express your answers in a tidy form.

2. (a) Write down, without proof, the rules for differentiating a product and for differentiating a function of a function (chain rule), i.e.

$$\frac{d}{dx}\left(f(x)g(x)\right), \quad \frac{d}{dx}f(g(x)),$$

   and state the value of

$$\frac{d}{dx}\left(\frac{1}{x}\right).$$

   From these facts alone derive the rule for differentiation of a quotient.

   (b) State the definition of the derivative of a function $f(x)$. From this definition derive the first derivative of the function

$$f(x) = x(1-x).$$

3. Consider a rectangular metal sheet with dimensions 10 metres by 6 metres. Squares of size $x$ metres by $x$ metres are cut out of the corners of the rectangular sheet (and thrown away) so that the rest of the sheet can be folded into an open top box. By means of a sketch show the construction described above.

   Find the expression for the volume of the final open top box as a function of $x$. Sketch a graph of this function.

   Determine the value of $x$ which gives the maximum box volume, and determine the volume in this case.

4. Suppose 1Kg of an unknown radioactive substance is buried for 10 years, and after this time is found to have mass 0.9Kg. If a simple exponential decay model is assumed determine the mass of the sample at the following times after burial:

   - 1 year,
   - 5 years,
   - 20 years.

   How long after burial would the sample have a mass exactly half of its initial value?

5. Evaluate the following definite integrals:

   (a) $\displaystyle\int_0^{2\pi} \sin(x/7)\,dx$,

   (b) $\displaystyle\int_1^{2} \ln(x)\,dx$,

   (c) $\displaystyle\int_0^{1} x\sqrt{1+x^2}\,dx$,

   (d) $\displaystyle\int_0^{1} x^2\exp(-x^3)\,dx$.

6. Using the trapezium method with 4 equal intervals find the approximate value of the following integral:

$$\int_0^1 \frac{1}{1+x}\,dx.$$

Write your approximation to the above integral as a rational number in its simplest form. By means of a sketch, illustrate the area that corresponds to the above integral, and state whether the trapezium method will give an under-estimate or an over-estimate in this case.

Hence or otherwise find an approximation to $\ln(4)$, stating clearly any properties of $\ln(x)$ that you use.

7. (a) Solve the following initial-value problem:

$$y' + y^2 = 0, \quad y(1) = 1.$$

(b) Find the general solution of

$$y'' + 4y' + 4y = x.$$