UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C394: Biomechanics

COURSE CODE	:	**MATHC394**
UNIT VALUE	:	**0.50**
DATE	:	**10-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

All questions may be attempted but only marks obtained on the best ***four*** *solutions will count.*
The use of an electronic calculator ***is*** *permitted in this examination.*

1. In each of the following state clearly any additional assumptions that you make, and where possible indicate how good you think that your assumptions are. Where necessary assume that the metabolic rate P of a mammal of mass M is given approximately by $P \propto M^{\frac{3}{4}}$.

   (a) Find the scaling laws of the following:

      (i) Heart rate.

      (ii) Respiration frequency.

      (iii) Oxygen consumption rate.

   (b) Consider an animal that lives on level ground, and determine how the time taken for this animal to dig a hole big enough for it to shelter in scales with its mass.

   (c) Explain briefly why, for a mammal, there is not a simple scaling law for the mass of the eye in terms of the body mass. What would be the scaling law of the mass of the skeleton of a land animal if static load were the only consideration, and why is the predicted mass too large?

   (d) How does the time taken for a biped to fall over when standing, if it does not correct its balance, scale with mass? Hence or otherwise determine how the minimum possible velocity of nerve impulses (between the brain and skeletal muscles) to enable dynamic balancing scales with mass.

2. (a) State in a few sentences some of the main adaptations for flight in most birds that are capable of flight.

(b) The equation for the drag is usually written in the following form:

$$D = D_f + D_i, \tag{1}$$

where

$$D_f = \tfrac{1}{2}\rho U^2 S C_{Df}, \quad D_i = KL^2/(\tfrac{1}{2}\rho U^2 b^2). \tag{2}$$

Define all the symbols used in the last equation. Sketch the function D against U.

(c) Develop the theory of the bounding flight of some small birds, determining (i) when bounding flight is possible, and (ii) under what conditions bounding flight is more efficient than continuously flapping flight. Find an expression for the optimal flapping fraction f for bounding flight.

(d) Show mathematically, using scaling arguments, that sufficiently big birds cannot fly.

3. (a) Describe the tracheal system of an insect in a few sentences.

(b) Write down the oxygen flux equations for both a pipe in the body network and a pipe in the gill network. Explain carefully the differences between the body and gill equations.

(c) Suppose that the oxygen concentration in air is ϕ_∞ and an aquatic insect is in a well oxygenated pond in equilibrium with the air over it. Consider a Y shaped gill network and a Y shaped body network, both of which have two blocked ends and a free end. Suppose further that there is no advection in either network and that all the pipes have the same cross-sectional areas, lengths ℓ, and oxygen exchange coefficients $\frac{1}{2}\nu^2$. If the gill network is now joined to the body network by connecting the free ends together, find expressions for the oxygen concentrations at all of the nodes in the networks (including the blocked ends).

What is the simplest expression for the oxygen concentration at the node joining the gill network to the body network in terms of ϕ_∞?

4. (a) Consider the motion of an elongated animal moving through a resistive medium in a limit where the tangent to any part of the animal's body is always close to the direction of motion. Let R_T and R_N be respectively the resistance per unit length to tangential and normal motion, and without making any assumptions about the magnitude of R_T/R_N show that

$$U \approx c\left(\frac{R_N - R_T}{R_N + \alpha^{-1}R_T}\right), \qquad (3)$$

where U is the speed of the animal with respect to the surrounding medium, c is the (phase) speed of the wave of body motion with respect to a frame where the animal has no net motion and α is a parameter which you should define.

(b) What values of R_T/R_N would be reasonable for

(i) a microscopic animal in water,

(ii) a snake moving through thick vegetation.

Apply the result of 4a to determine the speed of the animal in each of these cases.

(c) State briefly what improvements could be made to the model of this question.

5. (a) Describe the swimming motion of a bacterium on a microscopic scale. Under what conditions is the probability density function $p(\mathbf{r}, t)$ of one bacterium well approximated by

$$\frac{\partial p}{\partial t} = D\nabla^2 p,$$

where $\mathbf{r}$ is the position vector and t is time.

(b) Find the solution on $(-\infty, \infty)$ of

$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2},$$

with initial density function

$$p(x, 0) = \delta(x - a) - \alpha\delta(x + a),$$

where δ is a one-dimensional delta function and a, α are positive constants. State clearly any standard solutions of the diffusion equation that you use.

Show that the position $y(t)$ at which

$$p(y(t), t) = 0$$

moves at constant velocity u, and determine the value of u.

(c) Define the term *chemotaxis.* By using the results of the previous part of this question and a change of coordinates or otherwise, find the probability density function $p(x, t)$ on $x \geq 0$ for a bacterium with constant diffusivity D and constant drift velocity $u > 0$ starting at $x = a$, $a > 0$, with a killing boundary condition at $x = 0$ (i.e. a $p = 0$ boundary condition).

What is the probability that the bacterium is never killed by the hostile wall? Explain your reasoning carefully.