UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *B.Sc.(Econ)* *M.Sci.*

Mathematics M120: Algebra for Joint Honours Students

COURSE CODE	:	**MATHM120**
UNIT VALUE	:	**0.50**
DATE	:	**01-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0936-3-60

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Prove Lagrange's theorem, that the order of a subgroup H of a finite group G divides the order of G. (Any result appealed to must be stated clearly.)

   Using Lagrange's theorem, or otherwise, show that there are only two essentially different groups of order 4.

2. Let $\rho, \sigma, \tau \in S_9$ be

$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 9 & 8 & 2 & 1 & 5 & 7 & 6 & 4 \end{pmatrix}, \quad \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 8 & 7 & 3 & 9 & 2 & 5 & 6 & 1 \end{pmatrix},$$

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 6 & 9 & 2 & 3 & 8 & 5 & 4 & 7 \end{pmatrix}.$$

   Express ρ, σ, τ, ρ^{-1}, σ^2, $\rho\sigma^{-1}$, $\tau^{-1}\sigma$, $\tau\sigma\rho$ in cycle notation.

   Define the *signum* $\operatorname{sgn}\rho$ of a permutation $\rho \in S_n$. Prove that, if $\rho, \sigma \in S_n$, then $\operatorname{sgn}(\rho\sigma) = (\operatorname{sgn}\rho)(\operatorname{sgn}\sigma)$.

   Write down the signa of the 8 permutations ρ, $\ldots$, $\tau\sigma\rho$ above.

3. What relationship must hold between α and β so that the system of linear equations

$$\begin{array}{rcrcrcr} 2\xi_1 & + & 6\xi_2 & + & \alpha\xi_3 & = & -5 \\ \xi_1 & + & 3\xi_2 & - & 2\xi_3 & = & -2 \\ -2\xi_1 & - & 6\xi_2 & + & 3\xi_3 & = & \beta \end{array}$$

   has a solution? Find all the solutions when the equations are consistent.

   Explain, in terms of the augmented matrix of the system, exactly when a system of linear equations will have a solution.

4. What does it mean to say that a matrix is in *row echelon form*? Explain briefly why the row echelon form of an $m \times n$ matrix A over a field $\mathbb{F}$ is unique.

Reduce the following matrix (completely) to row echelon form:

$$\begin{bmatrix} 2 & 4 & 3 & -4 & -3 \\ -2 & -4 & -1 & 2 & -1 \\ -1 & -2 & -2 & 3 & 2 \\ -1 & -2 & -3 & 2 & 6 \end{bmatrix}.$$

5. Evaluate the determinant

$$\begin{vmatrix} 1 & 1 & 1 & 7 \\ -6 & 0 & 3 & -6 \\ 7 & -5 & -2 & 6 \\ -4 & 8 & -1 & 8 \end{vmatrix}.$$

Describe (with reasons) the effect on $\det A$, if a permutation σ is applied to the rows of the square matrix A. (Any results assumed must be stated clearly.)

6. Let

$$M := \begin{bmatrix} -3 & 1 & -1 \\ -4 & 1 & -2 \\ -2 & 1 & 0 \end{bmatrix}.$$

Either find an invertible matrix B and a diagonal matrix Δ such that $B^{-1}MB = \Delta$, or explain why no such matrices exist.