Answer THREE questions

1.

a) Describe the relationship between Markov chains and birth-death chains, and how they differ from other stochastic processes.

[4 marks]

b) What characterises a static Markov chain?

[3 marks]

c) What is meant by a steady state, and what are the conditions for a Markov chain reaching such a state?

[3 marks]

d) What is expressed by the Chapman-Kolmogorov equation?

[4 marks]

e) Derive an expression for the probability of having k births during the interval t for a pure birth chain with constant birth rate λ.

[11 marks]

Derive an expression showing how the intervals between births are distributed for a pure birth chain with constant birth rate.

[8 marks]

2.

a) Stochastic simulation models can be divided into finite-horizon and steady state models. What characterises the two types?

[4 marks]

b) Give examples of how horizons can arise

[3 marks]

c) Discuss in details the impact of an initial transient state on simulation results and possible ways of eliminating this influence.

[16 marks]

d) Discuss in details what criteria you would use to determine when to stop a simulation.

[10 marks]

3.

Consider a timesharing system containing a CPU and four devices (A, B, C and D) and a number of users.The probabilities for jobs completing at the CPU were found to be 4% to the users, 32% to device A, 44% to device C and 20% to device D. All jobs at device A are completing to Device B, and jobs at device C have a probability of 0.27 of completing to device B rather than directly to the CPU. Jobs at devices B and D complete directly to the CPU. The service times were 20 msec for device A, 15 msec for device B, 10 msec for device C and 14 msec for device D. The average service time per visit to the CPU was 5 msec, and the average user think time 5 seconds.

a) Create and label a diagram showng the system.

[6 marks]

b) For each job, what are the visit ratios for the CPU and each of the four devices (A, B, C and D)?

[6 marks]

c) What is the total service demand for the CPU and each of the four devices?

[8 marks]

d) What is the bottleneck device?

[6 marks]

e) If the utilisation of device A is 8%, what is the average response time if there are 10 users in the system?

[7 marks]

4.

a) Verify that 2 is a primitive element modolo 11.

[4 marks]

b) What conditions must a multiplicative congruential pseudo random number generator satisfy to ensure that it provides a full cycle?

[5 marks]

c) Consider the generators

$$X_{k+1} = 2\ X_i \mod 11$$

$$X_{k+1} = 5\ X_i \mod 11$$

and

$$X_{k+1} = 8\ X_i \mod 11$$

Which generator would you use to generate two-tuples, and why?

[8 marks]

d) Explain how a uniform random number generator can be used to generate random numbers which have a negative exponential distribution.

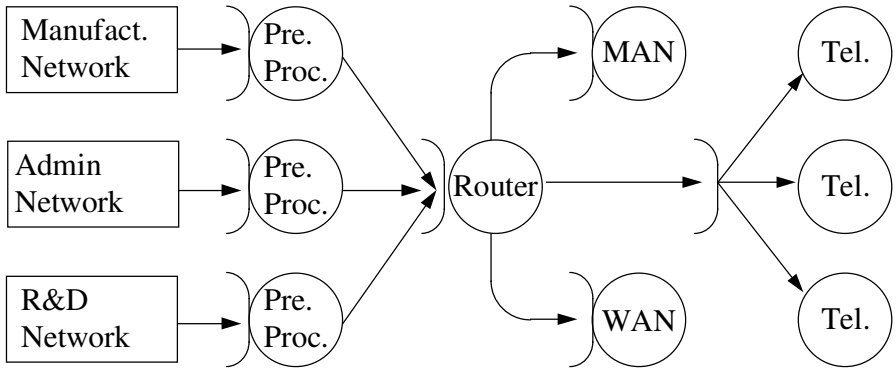
[8 marks]

e) Explain how a uniform random number generator can be used to generate random numbers which have a normal distribution.

[8 marks]

END OF PAPER

5.



The figure above shows elements of a data gateway used by the Fornda Motor Company. Only the section dealing with outgoing traffic is shown. Data is received from three separate internal networks serving: R&D, Administration and Manufacturing. Traffic from each network is pre-processed by a dedicated server before being queued for the router/switch for distribution to various external networks. All three pre-processors are identical with Poisson distributed service rate with a mean of 400 messages per second. They also have sufficient memory to provide an unlimited number of buffers. The service rate of the router is also Poisson distributed with a mean of 700 messages per second.

Messages from the three networks are Poisson distributed with mean rates of 165 messages per second from R&D, 350 from Administration and 185 from Manufacturing.

1. Determine the mean delay which messages from the Administration network suffer while waiting to be served by the pre-processor.

[10 marks]

2. Determine the mean waiting time and the mean number of messages in the queue if the queue in the router is limited to 36 buffers.

[11 marks]

One of the lines connected to the router is a point-to-point connection to another Fornda site. It consists of three load-sharing leased lines. Each line can be considered a Poisson server with a mean service rate of 15 messages per second.

3. What is the mean number of messages being served if the mean number of messages routed across this connection is 25 messages per second and the system can be considered to have an unlimited supply of buffers?

[12 marks]