B22 Models of Computation 1997

Answer any THREE questions from the five below

1. (a) What is a finite state machine? [10 marks]

(b) In principle, "finite" seems to mean that the machine cannot deal with any computation that has any features that are not instantly understandable as being finite, e.g. very long input strings whose lengths are not known. Is this belief right? What kinds of computation are not achievable by a finite state machine? Give an informal answer to this question.

[6 marks]

(c) For each of the languages below, decide if it can be recognised by a finite state machine. If it can be, draw a diagram for the machine that recognises it.

i) Over {a,b}, all the strings that end in baab.

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ii) Over {a,b}, all the strings that are palindromes (i.e. strings that are the same when read from left to right as from right to left).

[8 marks]

(d) Suppose that a light is controlled by two switches. Whatever is the status of the light (on or off) and whatever are the positions of each switch (up or down), moving one switch from one of its positions to the other changes the status of the light. Design and draw a finite state machine to represent this system.

[9 marks]

TURN OVER

2. (a) State Kleene's Theorem. What is its importance in the theory of computing?

[5 marks]

(b) Give an outline proof of either half of Kleene's Theorem.

[12 marks]

(c) State Cantor's diagonal argument.

[8 marks]

(d) Give an example of why Cantor's diagonal argument is useful in the theory of computing. What is the significance of this result, or of any other important result obtained via the same argument and referring to the theory of computing, to the subject?

[8 marks]

3. (a) What is a Turing machine?

[7 marks]

(b) Outline the design of a Turing machine that writes on its tape a copy of the string regarded as its input, but in the reverse order.

[9 marks]

(c) What is a universal Turing machine? Describe how such a machine could do its job, given a particular instance of the thing on which this job is to be done.

[6 marks]

[Question 3 cont. over page]

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[Question 3 cont.]

d) It is possible to define multi-tape Turing machines. What advantage might they have over single-tape Turing machines?

[5 marks]

(e) Give an informal answer to the question of whether there is any multi-tape universal Turing machine that can be transformed into a single-tape Turing machine that is not universal.

[6 marks]

4. (a) Define what is meant by a decision problem, and what it means to say that one problem P reduces to another problem P'.

[6 marks]

(b) State the Halting Theorem for Turing machines, and explain its importance in the theory of computing. (Hint: Don't forget part (a)).

[15 marks]

(c) Here is an old paradox. In a certain isolated monastery, one monk is designated to be the barber. The barber shaves all monks who do not have themselves. So, who shaves the barber?

Give an argument that has the same general character as the argument used in the proof of the Halting Theorem, to resolve this paradox.

[12 marks]

TURN OVER

5. (a) Define the terms alpha reduction, beta reduction and normal-order reduction, as they apply to the lambda calculus.

[6 marks]

(b) Reduce the following lambda-expression, where L stands for lambda, to its simplest form, using normal-order reduction.

(Lf . f (Lg . g (Lx . (f x)))) (Lx . x) (Lx . x) q

[10 marks]

(c) Are any other reduction orders possible with that expression? If so, do any of them give a different result? Would your answers to those questions depend on the particular lambda-expression involved, or would they always be the same because of some underlying property or properties of the lambda-calculus? Justify your claim.

[5 marks]

(d) Are lambda-calculus and functional programming two entirely separate concepts, or do they have some useful and/or technical connection with each other? Justify your answer.

[6 marks]

(e) Outline the design of a Turing machine that can apply one specifi reduction rule (of your choice) that occurs in the formulation of any of the reductions mentioned in part (a) of this question.

[6 marks]

END OF PAPER