Answer THREE questions, at least ONE from each section.

If you attempt MORE THAN THREE questions only the BEST THREE will be counted, but including ONE from each section.

Section 1. . *Answer at least ONE question.*

1. a) (i) Explain briefly the *divide and conquer* approach to algorithms.

(ii) Explain how quicksort works on the input <5, 3, 7, 1, 8, 9, 2 >

[10 marks]

b) Consider the problem of taking a list of numbers and a list of alphabet letters as input and returning a list of pairs, where the first pair in the list has the lowest ordered letter and the lowest ordered number and, in general, the i^{th} pair has the i^{th} lowest ordered letter and the i^{th} lowest ordered number. Assume the usual ordering on the numbers and the letters. Assume also that the list of letters is of the same length as the list of numbers. For example, input <a, f > and <3, 2 > gives the output <(a, 2), (f, 3) >.

(i) Give the pseudocode for an algorithm that can solve this problem. Consider how the lists can be represented in arrays and define the necessary subsidiary functions.

(ii) Show how the algorithm works for the input

<x, a, b, e, i, g, f > , <7, 3, 8, 9, 2, 1, 8 > .

(iii) This algorithm takes time in the order of what? Explain your answer.

[23 marks]

[Total 33 marks]

2. a) (i) What is a *recursive algorithm*? Explain briefly.

(ii) The following is a definition of for determining a Fibonacci number:

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Fib(n)
        if n < 2 then return n
                else return Fib(n - 1) + Fib(n - 2)
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Show that Fib(n) takes time of the order of Fib(n).

[13 marks]

[QUESTION 2 continues on next page]

[QUESTION 2 continued]

(b) A binary tree can be represented as a nested list, where a list is a triple containing a node and a list for each branch.

(i) Draw the binary tree corresponding to the following nested list:

[a, [b, [d, [], []], [e, [g, [], []], []]], [c, [], [f, [h, [], []], [i, [], []]]]] .

(ii) A nested list, such as in (i), was constructed using pre-order traversal. Construct another list for the same tree using in-order traversal.

(iii) Represent the tree in (i) as a directed graph (N, A), where N is a set of nodes and A is a set of directed arcs.

[15 marks]

(c) For the list of b(i) write down the sequence of nodes visited in a depth-first and in a breadth-first, search.

[5 marks]

[Total 33 marks]

3. (a) (i) Explain briefly the *greedy approach* to algorithms.

(ii) Explain briefly how Kruskal's algorithm can be used to find a minimum spanning tree.

[10 marks]

(b) A coin system is a set of coins of different integer values, $c_1,\ldots, c_n$, where $1 \leq c_1 \ldots < c_n$. For example, the British system is the set {1, 2, 5, 10, 20, 50, 100}. Consider the problem of giving change for a total value X using this currency system, so that the least number of coins is used. In other words, we need to find integers $k_1, k_2, \ldots, k_n$ such that

$$k_1c_1 + k_2c_2 + \ldots k_nc_n = X \text{ and } k_1 + k_2 + \ldots k_n \text{ is minimised.}$$

(i) Give the pseudocode for a greedy algorithm that solves this problem for a wide variety of coin systems.

(ii) Show that the algorithm works for the British coin system for giving change for 276p.

(iii) Give a coin system for which your algorithm does not work. Explain why it does not work.

[23 marks]

[Total 33 marks]

Section 2. ***Answer at least ONE question.***

4. The code characters $c_1, c_2, \ldots, c_N$ have probabilities $p_1, p_2, \ldots, p_N$ of occurrence respectively.

Explain Huffman's Method of encoding these characters into binary so as to minimise the average number of bits needed.

[14 marks]

Prove that the method works.

[14 marks]

The four characters a, b, c, d with probabilities p_1, p_2, p_3, p_4 are to be encoded. Under what circumstances would the Huffman method give an average length less than 2?

[5 marks]

[Total 33 marks]

5. *Explain carefully* what is meant by *order of convergence* when using an iterative method for solving an equation numerically.

[10 marks]

Newton's method for finding the square root of a positive number, a, is

$$x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k} \right)$$

for 'some suitable x_0'. What *would* be a suitable value? Justify your answer.

[11 marks]

Apart from Newton's method (above) here are two other iterative methods for finding the square root of a:

$$\text{i)} \quad x_{k+1} = \frac{2ax_k}{x_k^2 + a}$$

$$\text{ii)} \quad x_{k+1} = \frac{x_k(3a - x_k^2)}{2a}$$

Determine the order of convergence of these two methods. What are the advantages and disadvantages of these two methods?

{*You may find it helpful to know that* $(1 + z)^{-1} = 1 - z + z^2 - z^3 \ldots$}

[12 marks]

[Total 33 marks]

6. What do you understand by *heuristic methods* for the solution of a computational problem? Describe the points that have to be considered when applying such methods.

[8 marks]

You have been engaged as a consultant for Magic Supermarkets, who are planning the opening of a new shop. There are to be car parking facilities for customers. It is intended to have a car park with numbered spaces. The management is keen to have adequate space for its customers. It is accepted that on a small number of occasions the car park will be full, so that customers cannot come to the store. It is assumed that no customer stays for longer than 2 hours.

While the store is open two types of events can occur:

a) a car arrives and is parked in a randomly chosen space

or b) a randomly selected car departs.

You have been asked to advise on the appropriate size of car park for various probabilities of the car park being full.

Design an algorithm, in pseudo-code, with appropriate data structures, to help you in answering the management's question about car park size.

[25 marks]

Note: You are not required to write a program and are strongly advised not even to attempt to do so.

[Total 33 marks]

END OF PAPER