*Candidates should attempt **3** questions. If more questions are attempted only the best **3** will be counted. All questions carry the same marks.*

Wherever you are asked to design an algorithm it is acceptable for you to use properly annotated 'pseudocode', rather than a formal programming language.

1 Define a *minimal spanning tree* of a weighted undirected graph. Is such a spanning tree unique?

[5 marks]

Describe Prim's algorithm and Kruskal's algorithm for finding such a spanning tree.

[20 marks]

As both algorithms do the same task why should one bother with both of them?

[5 marks]

What happens to these two algorithms if the graph is not connected?

[3 marks]

2. Describe the data structure known as a *'B-tree of order n'*. Outline the advantages such a tree can be expected to have when searching for data records. What considerations might affect your choice of n?

[10 marks]

Explain how items may be inserted into such a tree or deleted from one. Illustrate the growth of a B-tree of order 2 when the items to be inserted are given in the order 2, 4, 3, 9, 5, 16, 7, 25, 11, 36, 13, 49, 17, 64, 19, 81, 23, 100, 29, 121, 31, 144, 37, 169, 41.

[15 marks]

If the tree has order n, how many records can be accessed altogether in a tree with k levels apart from the root? Estimate how many comparisons have to made to find a record furthest from the root.

[8 marks]

3 The median of a set of N numbers is the number that divides the set into 2 equally sized parts (with an appropriate adjustment to the definition if N is odd)., where all the numbers in one part are no larger than the median and in the other they are greater than the median.

Design an algorithm to calculate the median and *estimate* its efficiency.

[25 marks]

How would you modify your design to find the k^{th} smallest member of the set?

[8 marks]

4 What are *heuristic methods* in the context of solving an optimisation problem? Why might one wish to use such methods in preference to an 'exact' method?

[8 marks]

There has recently been a spate of thefts of computer memory from departments in UCL (and many other institutions). Improved security procedures are to be installed and the introduction of video cameras is being considered. You have been approached as a consultant to assist in determining the possible placement of such cameras. As cameras are quite expensive only a limited number of them can be placed in any department. A camera can cope with up to 3 straight corridors meeting at the position of the camera or 2 corridors and a flight of stairs to the next floor, again meeting at the position of the camera. The number of active cameras is to be kept to a minimum. Explain how the problem can specified in computational terms and indicate, in some, detail, how you might go about solving it. You are *not* required to give a complete program to solve the problem.

[20 marks]

It has been suggested that each corridor and flight of stairs should actually be covered by 2 cameras, to cover equipment failures. What modifications would be needed to your specification and what is the effect on your proposed method of attack?

[5 marks]

5 You are being asked to design a suite of procedures for the handling of polynomials,

$$p(x) = a_0x^n + a_1x^{n-1} + \ldots a_{n-1}x + a_n$$

(where the integer n is called the 'degree of the polynomial' and the terms a_i are the 'coefficients' which may be assumed to be real numbers). The procedures are to do the following tasks:

I) evaluation at $x = \alpha$;

ii) evaluation at $x = \alpha$ and $x = -\alpha$;

iii) adding two polynomials, not necessarily of the same degree;

iv) multiplying two polynomials, not necessarily of the same degree.

Design appropriate algorithms for these operations and indicate the data structures you would regard as most suitable.

[25 marks]

What modifications, if any, would you make if it is known that n is fairly large and many of the coefficients are 0?

[8 marks]

END OF PAPER