2.

a) In algorithmic analysis, what is meant by the term **elementary operation**? Why is cost best measured in terms of elementary operations rather than in CPU time?

[6 marks]

b) Two algorithms A and B solve the same algorithmic problem. In the worst case A's solution has a lower order of complexity than B's, but on average the situation is reversed. Should either of these algorithms be considered 'better' than the other? If yes, explain why; it no, describe circumstances under which each of the algorithms might be preferred.

[6 marks]

c) Are the following statements true or false? Justify your answers using a careful argument based on the mathematical definition of 'O' notation.

(i) $n^2 \in O(n^3)$

(ii) $n + \frac{1}{n} \in O(n)$

(iii) $(n+1)! \in O(n!)$

[6 marks]

d) Describe how the efficiency of a basic Quicksort procedure can be improved by some simple extensions to the algorithm. Give an idea of the reduction in running time (on most applications) that might be expected by adopting these modifications to the algorithm.

[6 marks]

e) Consider the following recurrence relations (where a and b are constants):

$$f(0) = a$$
$$f(1) = b$$
$$f(n) = 2\,f(n-1) - f(n-2), \quad n > 1$$

$$g(1) = 6$$
$$g(2) = 10$$
$$g(n) = 3\,g(n/2) - 2\,g(n/4), \quad n > 2$$

Solve each of these recurrence relations as a function of the variable n (and, where relevant, a and b). What values must a and b have in order that f(n) takes *exactly* half the time that g(n) takes, for all comparable input sizes n?

[9 marks]

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