Electronic calculators are permitted.

Note : where an algorithm is asked for, you may write in any suitable pseudo-code. Correct syntax for any computer language is not expected.

1. a) Discuss the difference between a vector and a raster output device

[4 marks]

b) Describe the basic components and method of operation of a monochrome cathode ray tube (CRT). Include in your answer a definition of the terms *spot size*, *addressability*, *persistence*, and *refresh rate*.

[15 marks]

c) What are the fundamental differences from your answer to b) in the operation of a colour CRT ?

[4 marks]

d) Briefly describe two ways in which the design of these display devices is dictated by our understanding of the mechanism of human visual perception

[5 marks]

e) Why is a CRT normally operated in raster mode ? Explain the benefits and disadvantages of operating in vector mode for both a monochrome and colour CRT.

[5 marks]

TURN OVER

Cont...

2. a) Explain the use of windows and viewports in defining the coordinate systems used in Computer Graphics.

[5 marks]

b) Explain how a *partitioning function* can be specified to define general lines or curves in 2D space, or surfaces in 3D space. Show how this gives rise to a directedness of these objects.

[6 marks]

c) If a window is specified by (WX_{min}, WY_{min}, WX_{max}, WY_{max}) give the partitioning functions $\{f_{left}(x,y), f_{bottom}(x,y), f_{right}(x,y), f_{top}(x,y)\}$ of the bounding edges, and the *line-normals* $\{\hat{\boldsymbol{n}}_{left}, \hat{\boldsymbol{n}}_{bottom}, \hat{\boldsymbol{n}}_{right}, \hat{\boldsymbol{n}}_{top}\}$ to these edges (i.e. the vectors perpendicular to these edges).

[4 marks]

d) Consider a finite line segment described by the parametric equation

$$\mathbf{P}(\lambda) = \mathbf{P}_1 + \lambda(\mathbf{P}_2 - \mathbf{P}_1)\ ,\ \lambda \in [0,1] \qquad (1)$$

Let $s_i \triangleq \hat{\boldsymbol{n}}_i.(\mathbf{P}_2-\mathbf{P}_1)$ ($i \in$ {left, bottom, right, top}) be the set of four scalar products of this line with the line normals derived in part c).

Show that for any line described by Eq. (1) either two s_i will be positive and two negative, or two will be zero and the other two equal and opposite.

[4 marks]

e) By means of a diagram, or otherwise, find the four values $\{\lambda_{left}, \lambda_{bottom}, \lambda_{right}, \lambda_{top}\}$ that give the intersection of the infinite line containing the line segment described in Eq. (1), with the four window boundaries.

[4 marks]

f) Using the notation introduced above, describe the *Liang-Barsky* algorithm for clipping a line against an upright rectangular window. Illustrate exactly the working of the algorithm for a window given by ($WX_{min} = 10$, $WY_{min} = 10$, $WX_{max} = 20$, $WY_{max} = 20$) and the lines given by:

i) $\mathbf{P}_1 = (0,10)$, $\mathbf{P}_2 = (15,35)$ ii) $\mathbf{P}_1 = (12,14)$, $\mathbf{P}_2 = (20,6)$

[7 marks]

g) Indicate how the Liang-Barsky algorithm could be extended to clipping against any convex polygon

[3 marks]

3. a) What is the Homogeneous Coordinate Representation? Explain with reference to its use in both 2D and 3D graphics.

[8 marks]

b) Show that a general transform of the form

$$A = \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ e & f & 1 \end{pmatrix}$$

can be expressed as a transform that preserves the origin followed by a translation.

[2 marks]

c) When does A have an inverse ? In the case where the inverse exists, give it explicitly. [Hint : consider your solution to part b)]

[4 marks]

d) By considering the transformation of the unit square, show graphically how two-dimensional Cartesian space is transformed under A. Indicate as a special case what happens when A does not have an inverse.

[8 marks]

e) If A does not have an inverse what can you say about solutions to the equation:

$$[\mathbf{x}]\ A = [\mathbf{b}]$$

where [**x**] and [**b**] are row-vectors ?

[3 marks]

f) Explain how your answer to part e) is related to the problem of converting 3D object-space to 2D image-space, with particular reference to ray-tracing.

[8 marks]

Cont...

4. a) Define the terms *surface normal* $\hat{\boldsymbol{n}}$, *eye vector* $\hat{\boldsymbol{e}}$, *light vector* $\hat{\boldsymbol{L}}$, and *reflection vector* $\hat{\boldsymbol{R}}$ as they are applied in the lighting model used in Computer Graphics. Show how $\hat{\boldsymbol{R}}$ can be derived from $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{L}}$.

[6 marks]

b) Define the terms *ambient*, *diffuse*, and *specular* as used in the lighting model. Discuss how objects may be given different material attributes so as to vary the relative contributions of these three terms.

[9 marks]

c) Consider an isolated polygon defined by the four points {(-10,-10,0), (10,-10,0), (10,10,0), (-10,10,0)}, a point light source situated at (-5,0,5), and a camera situated at (5,0,5) with view normal in the direction (-1,0,-1). By means of a sketch diagram or otherwise, indicate the variation of $\hat{\boldsymbol{n}}$, $\hat{\boldsymbol{e}}$, and $\hat{\boldsymbol{L}}$, and the ambient diffuse and specular light intensity terms, along the line from (-10,0,0) to (10,0,0).

[6 marks]

d) Repeat the answers to part c) for the case where the light is now situated at (-500,0,500), and the camera at (500,0,500).

[3 marks]

e) Suppose that the polygon defined in part c) is now part of a mesh of approximately equally sized polygons representing the surface of a sphere with centre (0,0,-100) and radius 100, with the light and camera at the positions given in part d). Estimate the number of polygons in the representation, and by means of further sketches or otherwise, indicate the variation of the ambient diffuse and specular terms along the same line as in part c), for the following rendering methods :

i) Constant shading
ii) Brute Force shading
iii) Gouraud shading
iv) Phong shading.

Compare your answers with those that would be obtained from a ray-tracing rendering of a sphere defined explicitly by the quadratic equation

$$x^2 + y^2 + (z+100)^2 = 100^2$$

[9 marks]

5 (a) The diagram below represents a set of 3D planar "polygons" in the usual way. The arrows indicate the "front" of each polygon. Construct a BSP tree for this set of polygons, where the BSP tree itself contains the minimal number of polygons which is possible.

[6 marks]

(b) The point c is the centre of projection. Assuming back-face elimination, what polygons will an observer at c actually be able to see?

[6 marks]

(c) Use the BSP tree traversal algorithm to obtain the back-to-front order of polygons from position s.

[7 marks]

(d) Suppose there is a point light source at position s. Describe in detail how polygons corresponding to shadows caused by this light source would be computed.

[7 marks]

(e) Compare the BSP hidden surface algorithm with the z-buffer algorithm, stating the advantages and disadvantages of each.

[7 marks]

END OF PAPER