University College London

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

M.Res

COURSE CODE	:	**MMAI**
TITLE OF EXAMINATION	:	**Mathematical Methods, Algorithms and Implementations**
DATE	:	**3-MARCH-1997**
TIME	:	**09.30**
TIME ALLOWED	:	**2 hours 30 minutes**

TURN OVER

Open Book Exam : You may bring lecture notes and textbooks into the exam

Answer two questions from part A, and two questions from part B

PART A

1) a) Draw a functional block diagram for a computer inspection system that is used to reject broken digestive biscuits.

[4 Marks]

b) Is this a real-time system, and if so why?

[2 Marks]

c) Describe the basic processing steps required.

[5 Marks]

d) Estimate the maximum rate at which the system could be run.

[8 Marks]

e) Why is it disadvantageous to have a bright background when using a CCD camera. Describe the effect of a bright background on the image, and why it occurs.

[3 Marks]

f) Explain how a DMA device might be used in this system.

[3 Marks]

2) Computer architecture.

a) Which parameters should remain in the processors internal registers, and why?

[2 Marks]

b) Describe the three most common special purpose internal registers and their function. [3 Marks]

c) Write a few lines of C code in which the stack pointer is changed

[3 Marks]

d) How do the Harvard and von Neuman architectures differ? What are the advantages of each? [5 Marks]

e) What is partial address decoding, and why is it used? What occurs if an undecoded region of memory is read? [6 Marks]

f) What is tri-state logic and why is it important in the design of digital systems?

[6 Marks]

3) Interrupts

a) What steps are taken by a 68000 family microprocessor after it receives an interrupt?

[7 Marks]

b) How does the processor distinguish between two devices that use the same interrupt level?

[4 Marks]

c) When you get the error message "segmentation violation" what has occurred?

[4 Marks]

d) In what way are interrupts used to balance time between tasks in a multi-user operating system?

[10 Marks]

PART B:

4) a) A signal f(t) is sampled from -T/2 to T/2 in N samples. What frequencies in f(t) will be aliased ?

[2 Marks]

b) Suppose f(t) is a Lorentzian : $1/2\ \alpha\ \mathrm{Exp}[-\alpha|t|]$. If $T = 4/\alpha$ and N = 8, estimate the fraction of signal outside the sampling interval in both the temporal and Fourier domains. Show how your answer is changed if

i) T is doubled and N remains constant

ii) T remains constant and N is doubled

iii) Both T and N are doubled

[10 Marks]

c) It is proposed to use a Lorentzian to prefilter (by convolution) the signal before sampling. Draw diagrams to illustrate the effect of the filter in the temporal and Fourier domains. What value of α would be appropriate to ensure that all frequencies above the Nyquist frequency are reduced by a factor of 100 or more.

[6 Marks]

d) How could the filter of part c) be implemented in a recursive manner ?

[4 Marks]

e) Comment on the suitablity or otherwise of the filter suggested in part c)

[3 Marks]

[**CONTINUED**]

5) a) Define $L_w(\sigma)$ as the derivative in the w-direction of an image convolved with a Gaussian width σ. Show how to calculate this quantity in the Fourier domain, for both w=x and w=y.

[4 Marks]

b) Define a local coordinate system (u,v) as a pair of orthogonal unit vectors making an angle θ with the x and y axes. Show that :

$x = u\,\mathrm{Cos}\theta - v\,\mathrm{Sin}\theta,$

$y = u\,\mathrm{Sin}\theta + v\,\mathrm{Cos}\,\theta$

and thus find expressions for the derivative operators d/du and d/dv.

[6 Marks]

c) In terms of the local coordinate system of part b), curvature is defined as $L_{vv}(\sigma)/L_u(\sigma)$, where u is taken along the direction of the image gradient, and v is perpendicular to this direction. Show that the curvature is equal to

$$(L_x^2(\sigma)L_{yy}(\sigma) + L_y^2(\sigma)L_{xx}(\sigma) - 2L_x(\sigma)L_y(\sigma)L_{xy}(\sigma))/(L_x^2(\sigma) + L_y^2(\sigma))^{3/2}$$

[10 Marks]

d) Give a description of a method to calculate curvature efficiently in the Fourier domain, and compare with any other method for calculating curvature that you know.

[5 Marks]

6) A *balloon model* represents a simulated thin-plate material under tension. It is represented as a vector-valued parametric function :

$$\mathbf{x}(u,v) = [x(u,v),\ y(u,v),\ z(u,v)]^T$$

where vector **x** represents the positions of material points (u,v) relative to a reference frame in Euclidean 3-space. The deformation energy of the thin-plate under tension is given by the energy functional :

$$\varepsilon_p(\boldsymbol{x}) = \iint \alpha_{10}\left|\frac{\partial \boldsymbol{x}}{\partial u}\right|^2 + \alpha_{01}\left|\frac{\partial \boldsymbol{x}}{\partial v}\right|^2 + \beta_{20}\left|\frac{\partial^2 \boldsymbol{x}}{\partial u^2}\right|^2 + \beta_{11}\left|\frac{\partial^2 \boldsymbol{x}}{\partial u \partial v}\right|^2 + \beta_{02}\left|\frac{\partial^2 \boldsymbol{x}}{\partial v^2}\right|^2 \mathrm{d}u\mathrm{d}v$$

a) Describe the physical meaning of each of the constraint terms in this equation.

[5 Marks]

[Question 6 continued over page...]

[Question 6 continued...]

b) The balloon may be used to fit a closed continuous surface within a 3D volume of voxels. The dynamic behaviour of the balloon model during the fitting process is governed by the second-order partial differential equation

$$\mu\frac{\partial^2 \boldsymbol{x}}{\partial t^2} + \gamma\frac{\partial \boldsymbol{x}}{\partial t} + \varepsilon_p = \boldsymbol{f} \qquad \textbf{(2)}$$

where μ(u,v) represents inertial force, γ(u,v) represents damping forces, and **f** represents external forces derived from the image data. By discretising **x** into a suitable basis, show that Eq.(2) can be represented as a matrix equation

$$\mathrm{M}\frac{\partial^2 \boldsymbol{q}}{\partial t^2} + \mathrm{C}\frac{\partial \boldsymbol{q}}{\partial t} + \mathrm{K}\boldsymbol{q} = \boldsymbol{f}_q \qquad \textbf{(3)}$$

and give the explicit form of the three matrices (M,C and K). Discuss how you might solve Eq.(3).

[20 Marks]

[END OF PAPER]