There are five questions on this paper. Answer THREE questions.

1.
Consider the curve defined by the following rational polynomials:

$$x(t) = \frac{1 + 3t + 6t^2 + 2t^3}{1 + 6t - 3t^2}, \quad y(t) = \frac{3 + 12t^2 - 3t^3}{1 + 6t - 3t^2}, \quad t \in [0,1]$$

(a) Find the polar forms for each of the three polynomials given.

[5 marks]

(b) Find the control points and weights for the representation of this curve in the cubic rational Bezier form.

[10 marks]

(c) Without explicitly evaluating any derivatives, find control points for a second cubic rational Bezier curve over the domain [1,b] that joins to this one with C^2 continuity at the knot t = 1.

[10 marks]

2. The following is based on the sequence of cubic B-Spline control points (0,0), (1,0), (1,1), (0,1), (0,2).

(a) Construct a uniform knot vector, starting at 0 with interval 1, but adjusted so that the B-Spline curve starts at the first control vertex, and ends at the last.

[5 marks]

(b) How many piecewise cubic polynomials does this B-Spline consist of and what are their parameter domains?

[5 marks]

(c) Describe how to use the Oslo Algorithm to simultaneously insert two further knots of $u_1 = 1.2$ and $u_2 = 1.4$, and show how to compute the resulting sequence of control points. (You do not need to work through all the numerical calculation).

[10 marks]

(d) Suppose there is a two-dimensional array of B-Spline 3D control points v_{ij} (i = 0,...,m; j = 0,...,n). In general, outline how to compute a point on the cubic B-Spline approximating surface for these control points, given suitable knot sequences. (Your answer should specify, for the general case, the appropriate knot-sequences).

[5 marks]

3.

(a) The equation $x^2 + y^2 - z^2 = 0$ defines a cone. A ray starts at p_0 in the direction dp. Find the intersections of the ray, if any, with the cone.

[5 marks]

(b) Suppose that p_0 is on another surface, and the normal at p_0 is n_0 . Find the differential form-factor between the small area around p_0 and the small area around any point $p = (x,y,z)$ on the cone.

[8 marks]

(c) How would you make use of this result to find an approximation to the form-factor relating the differential area around p_0 to that part of the surface of the cone which is visible at p_0 ?

[7 marks]

(d) In general how would you approach the problem of doing the radiosity computation on a scene constructed entirely from quadric surfaces?

[5 marks]

4.

(a) Describe the approach to global illumination known as ray tracing, outlining the main ray tracing algorithm.

[5 marks]

(b) Outline the main approaches for reducing the number of ray-intersection tests.

[5 marks]

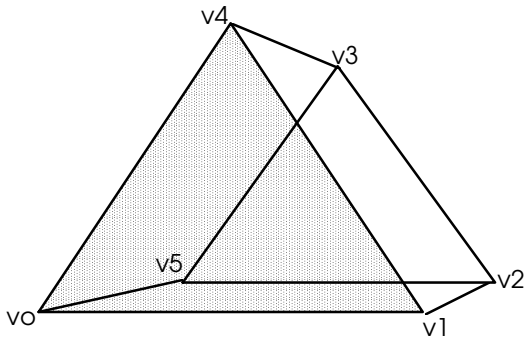
(c) Describe in detail the ray classification method of due to Arvo and Kirk.

[15 marks]

5. Consider the shape below to be represented using the Winged Edge data structure.

(a) For edges 0 and 4 indicate the Next Vertex, Previous Vertex, Next Face and Previous Face for the Winged Edge data structure.

[4 marks]



$v_0\ v_1\ v_4 = Face_0$
$v_5\ v_3\ v_2 = Face_1$
$v_1\ v_2\ v_3\ v_4 = Face_2$
$v_0\ v_4\ v_3\ v_5 = Face_3$
$v_0\ v_5\ v_2\ v_1 = Face_4$

$edge_0 = v_0\ v_1$
$edge_1 = v_1\ v_2$
$edge_2 = v_2\ v_5$
$edge_3 = v_5\ v_0$
$edge_4 = v_1\ v_4$
$edge_5 = v_2\ v_3$
$edge_6 = v_5\ v_3$
$edge_7 = v_0\ v_4$

(b) Explain how the winged edge data structure would be completed - that is how the wings would be found. Give an example using edges 0 and 4.

[6 marks]

(c) What is meant by each the following terms in the context of representations of 3D polyhedral objects, and why are they important?

- level of detail
- progressive mesh representation.

[5 marks]

(d) Outline Hoppe's (1996) method for progressive meshes, discussing the advances claimed for this method.

[10 marks]

END OF PAPER