1.

(a) The following is the blossom of a bi-cubic patch defined over parametric values $0 \le t,u \le 1$:

$$f_x(t_1,t_2,t_3;u_1,u_2,u_3) = 1 + u_1 + u_2 + u_3 + (t_1 + t_2 + t_3)(u_1u_2 + u_1u_3 + u_2u_3) + t_1t_2t_3u_1u_2u_3$$

$$f_y(t_1,t_2,t_3;u_1,u_2,u_3) = 2 + t_1t_2t_3(u_1u_2 + u_1u_3 + u_2u_3) + u_1u_2u_3$$

$$f_z(t_1,t_2,t_3;u_1,u_2,u_3) = 3 + u_1u_2u_3(t_1t_2 + t_1t_3 + t_2t_3) + t_1t_2t_3$$

This blossom corresponds to a bi-cubic Bezier surface $P(t,u)$, with control points $p_{ij}, i, j = 0,1,2,3.$

Find an expression for $P(t,u)$ in the power basis. Find p_{00}.

(b) For a triangular Bezier patch of degree 3, describe in detail the arrangement of control points, and also how to render such a patch using a de-Casteljau subdivision algorithm.

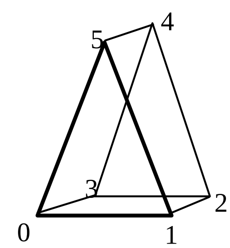
(c) A cubic B-Spline curve has equally spaced knots, and one particular segment of the curve has B-Spline control points (0,0), (1,0), (1,1), and (0,1). Insert a new knot half-way between the two central knots, and find the new control points.

2.

(a) Describe the winged-edge data structure – its motivation, its components, and the typical operations for which it is used.

(b) The diagram below shows a wedge-shape figure. Each face is outward facing. Draw a diagram and a corresponding explanation, of how the various elements of the structure link together. Also describe how the 'wings' are formed. Use the following naming conventions:

Edge Number	Vertices	Face	Vertices
0	0,1	Near	0,5,1
1	1,2	Right	1,5,4,2
2	3,2	Far	3,4,2
3	0,3	Left	0,5,4,3
4	0,5	Bottom	0,3,2,1
5	1,5		
6	2,4		
7	3,4		
8	5,4		



(c) Suppose that each face of a polyhedra has a corresponding normal vector. The faces sharing a common vertex are to have their normals averaged to produce an average normal at the vertex. Describe how the winged edge data structure could be used for this purpose.

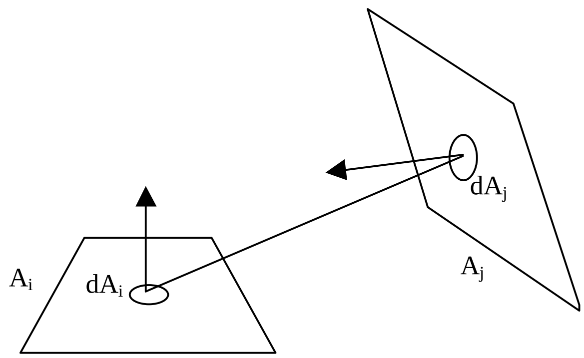
3.

(a) Write down Kajiya's 'rendering equation' and explain its components.

(b) Show how the solution to the rendering equation 'unfolds' into an infinite series, and explain the computer graphics interpretation of each successive term in the series.

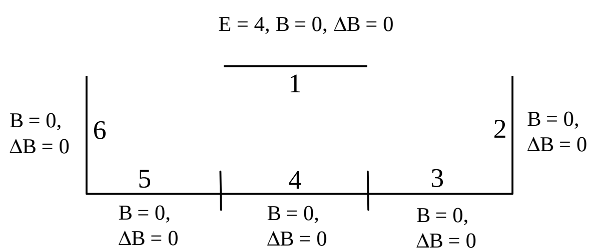
(c) Describe in detail a method of implementation that can for any actual scene solve the rendering equation.

4.



(a) Given the configuration above, where A_I and A_j are patches and dA_i and dA_j are differential areas on these patches, give the formulas for the form factors F_{dAidAj} (between the differential elements dA_i and dA_j), F_{dAiAj} (between differential element dA_i and patch A_j) and F_{AiAj} (between the two patches). Explain what is the meaning of the form factor in the context of radiosity. In the figure below the table on the right gives the form factors between pairs of patches. Why do the form factors of patch 4 not add up to 1, ($\Sigma F_{4*} < 1$)? What does this imply in terms of the outgoing radiosity of patch 4?

(b) Describe two common ways for computing the form factors.



length of each patche = 1
ρ for all patches = 0.7
Emmitance = 0 except patch 1

i \ j	1	2	3	4	5	6
1	0	.1	.2	.4	.2	.1
2	.1	0	.3	.1	.05	.1
3	.2	.3	0	0	0	.05
4	.4	.1	0	0	0	.1
5	.2	.05	0	0	0	.3
6	.1	.1	.05	.1	.3	0

(c) Assuming monochromatic light (only one wavelength), give the values of ΔRad and Rad after two iterations of progressive refinenment radiosity for the set-up of the figure above, explaining your calculations.

5.

(a) Explain how particle tracing works. Include in your answer how particles are chosen, how they are traced and how the final result is displayed.

(b) How can the technique called “Russian Roulette” be used for accelerating particle tracing without introducing new errors?

(c) Under what circumstances will a particle tracing solution give results similar to radiosity? When will one be preferable to the other? Under what circumstances will it give results similar to ray-tracing?