# MAT2011 Spring 2011: Unassessed Homework Assignment #3

---

**3.1** Assume that $g(x)$ is continuous and has period $2\ell$. Prove that

$$\int_{-\ell}^{\ell} g(x)\,\mathrm{d}x = \int_{-\ell+a}^{\ell+a} g(x)\,\mathrm{d}x$$

is independent of $a \in \mathbb{R}$. In particular, it does not matter over which interval the Fourier coefficients are computed as long as the interval length is $2\ell$. *[Remark: This result is also true for piecewise continuous functions].*

---

**3.2** Consider the function $f(x)$ defined via

$$f(x) = \begin{cases} 1 & 0 \le x < 1 \\ 2 & 1 \le x < 3 \end{cases}$$

and extended periodically with period 3 to $\mathbb{R}$ so that $f(x+3) = f(x)$ for all $x$.

(i) Find the Fourier series of $f(x)$.

(ii) Discuss its limit: In particular, does the Fourier series converge pointwise or uniformly to its limit, and what is this limit?

(iii) Plot the graph of $f(x)$ and the limit of the Fourier series.

---

**3.3** Consider the sequence $f_n(x) = (1-x)x^{n-1}$ for $x \in [0, \frac{1}{2}]$. Prove that the series $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly to $f(x) = 1$ on $[0, \frac{1}{2}]$.

---