MS2011 Spring 2011: Unassessed Homework Assignment #2

This sheet is for practice only. If you get stuck then please refer initially to the solutions provided on the website. Further examples can be found in Chapter 6 of the Guidance Notes.

6.1 Using the method of characteristics, solve

$$4u_x - 3u_y = 0$$

subject to

$$U(x, 0) = x^2.$$

Where in $\mathbb{R}^2$ is your solution valid?

6.2 Sketch the characteristics of the PDE

$$(1 + x^2)u_x + u_y = 0$$

and write down its general solution.

6.3 Sketch the characteristics of the PDE

$$xu_x + 2yu_y = 0$$

and write down its general solution.

For each of the following boundary conditions either solve the PDE completely, or, if this is not possible, say why the method of characteristics fails.

(i) $u(1, y) = e^y$ for $y > 0$;

(ii) $u(x, 0) = h(x)$ for $x \in \mathbb{R}$.

If you do find a solution you should state clearly where it is valid.

6.4 By using the substitution $u = e^x v$, or otherwise, write down the general solution to the PDE

$$u_x + 2u_y = u.$$
