DEPARTMENT OF PHYSICS & ASTRONOMY

Autumn Semester 2006-2007

OPTICAL PROPERTIES OF SOLIDS **2 Hours**

Answer THREE QUESTIONS.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

NOTE

In this examination you may assume without proof that the relationship between the complex refractive index of a material $\overline{n} = n + i\kappa$ and its complex relative dielectric constant $\tilde{\varepsilon}_r = \varepsilon_1 + i\varepsilon_2$ is given by

$$n = \frac{1}{\sqrt{2}}\left[\varepsilon_1 + \left(\varepsilon_1{}^2 + \varepsilon_2{}^2\right)^{1/2}\right]^{1/2}$$

$$\kappa = \frac{1}{\sqrt{2}}\left[-\varepsilon_1 + \left(\varepsilon_1{}^2 + \varepsilon_2{}^2\right)^{1/2}\right]^{1/2} .$$

1 (a) The medium through which a wave (eg a water, sound or a light wave) propagates is sometimes described as being *dispersive*. What does this mean? **[1]**

(b) Explain what is meant by *normal* dispersion in optics, and account for the fact that most glasses that are transparent at optical frequencies have normal dispersion in the visible spectral range. **[2]**

(c) In what spectral regions would you expect to observe *anomalous* dispersion in a typical transparent glass? **[1]**

(d) Most glasses are made predominantly from silica (SiO_2), but some glass manufacturers add lead oxide, which has a smaller band gap than SiO_2, to increase the sparkle of the glass. Why does the addition of lead oxide increase the reflectivity of the glass? **[1]**

(e) (i) Explain why the time taken for a wave packet at angular frequency ω to propagate through a dispersive medium of length L can be written in the form

$$t = \frac{L}{\mathrm{d}\omega/\mathrm{d}k} = L\frac{\mathrm{d}k}{\mathrm{d}\omega} .$$

By writing $\frac{\mathrm{d}k}{\mathrm{d}\omega} = \frac{\mathrm{d}k}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}\omega}$, show that the propagation time for a light pulse is given by

$$t = \frac{L}{c}\left(n - \lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right),$$

where n is the refractive index and λ is the vacuum wavelength. **[2]**

(ii) A short laser pulse propagates through a dispersive medium such as an optical fibre. By using the result in part (i), show that the temporal broadening of the pulse is given by

$$\Delta\tau = \frac{L}{c}\left|\lambda^2\frac{\mathrm{d}^2 n}{\mathrm{d}\lambda^2}\right|\frac{\Delta\lambda}{\lambda} ,$$

where $\Delta\lambda$ is the spectral band width of the pulse. **[1]**

(iii) A pulse of wavelength 1550 nm and initial duration 1 ps is launched into a silica optical fibre of length 100 m. Calculate $\Delta\tau$ for this pulse, given that $\lambda^2\ \mathrm{d}^2 n/\mathrm{d}\lambda^2 = -0.01$ at 1550 nm for silica. **[1]**

(iv) Standard silica optical fibres have $\mathrm{d}^2 n/\mathrm{d}\lambda^2 = 0$ for wavelengths around 1300 nm. How is this possible? **[1]**

2 (a) Sketch the absorption spectrum that you would expect to observe near the band gap at room temperature for:

(i) a direct gap semiconductor such as InAs with a small exciton binding energy (i.e. □ $k_B T$, where T = temperature);

(ii) a direct gap semiconductor such as GaN with a large exciton binding energy (i.e. $\geq k_B T$);

(iii) an indirect gap semiconductor such as Si. **[3]**

(b) How would you expect the band edge absorption spectrum of silicon to change as the temperature is decreased from room temperature to 4 K. **[1]**

(c) The band structure parameters of the direct gap semiconductor InAs at 300 K are as follows:

Band gap energy $E_g = 0.35$ eV

Spin-orbit splitting energy $\Delta = 0.38$ eV

Electron effective mass $m_e^* = 0.022\ m_0$

Heavy hole effective mass $m_{hh}^* = 0.4\ m_0$

Light hole effective mass $m_{lh}^* = 0.026\ m_0$

Split-off hole effective mass $m_{so}^* = 0.14\ m_0$

Calculate the k vectors of the final electron states after

(i) a heavy hole transition at 0.4 eV,

(ii) a light hole transition at 0.4 eV,

(iii) a split-off hole transition at 0.8 eV. **[3]**

(d) The absorption edge of a sample containing InAs quantum dots is found to occur at 1000 nm at room temperature.

(i) Explain what is meant by a *quantum dot*.

(ii) On the assumption that the dots have a cubic shape, estimate the size of the dots. **[3]**

3 (a) Explain what is meant by the *radiative quantum efficiency* of a light-emitting device. **[1]**

(b) The lifetime of the upper laser level in a Co:MgF_2 crystal is equal to 1.4 ms at 77 K and 0.06 ms at 300 K.

(i) Calculate the radiative quantum efficiency at the two temperatures, given that the radiative lifetime of the transition is equal to 1.8 ms.

(ii) Suggest a reason why the lifetime is shorter at 300 K than at 77 K. **[2]**

(c) Explain why light emitting diodes made from direct gap semiconductors are generally more efficient than those made from indirect gap semiconductors. **[1]**

(d) Most laser diodes made nowadays incorporate quantum wells in the active region. Why is this? **[1]**

(e) The band gap of GaAs at room temperature is 1.42 eV, and the electron and heavy hole effective masses are 0.067 m_0 and 0.5 m_0 respectively. A p-i-n diode contains GaAs quantum wells with a width of 10 nm in the i-region.

(i) Estimate the emission wavelength that would be observed when the diode is operated in forward bias.

(ii) How would you expect the photoluminescence spectrum to vary with the voltage when the device is operated in reverse bias? **[3]**

(f) An n-type intersubband laser is made with the same quantum wells as used in part (e). Estimate the emission wavelength of the device. **[2]**

4 (a) Show that the frequency dependence of the relative dielectric constant of a gas of undamped free electrons is given by

$$\varepsilon_{\mathrm{r}}(\omega) = 1 - \frac{Ne^2}{\varepsilon_0 m_{\mathrm{e}} \omega^2} ,$$

where ω is the angular frequency and N is the number of electrons per unit volume. **[3]**

(b) Use the formula in part (a) to explain why metals are expected to have near 100% reflectivity for frequencies below a certain characteristic frequency called the plasma frequency, ω_{p}, and derive a formula for ω_{p}. **[2]**

(c) Account qualitatively for the fact that the measured reflectivity of a metal like aluminium is only about 90% in the visible spectral region, even though the plasma frequency corresponds to a wavelength in the vacuum ultraviolet spectral region. **[1]**

(d) Potassium metal is found to be transparent for wavelengths shorter than 315 nm.

(i) Use this information to estimate the electron density in potassium.

(ii) The actual electron density in potassium is equal to 1.4×10^{28} m^{-3}. Compare this to the value calculated in part (i), and account for any discrepancy. **[2]**

(e) Explain what is meant by a *plasmon*, and discuss how plasmons might be observed experimentally in:

(i) a metal like aluminium,

(ii) a heavily-doped n-type semiconductor. **[2]**

5 (a) The frequency dependence of the relative dielectric constant of an ionic crystal in the infrared spectral region is given by

$$\varepsilon_{\mathrm{r}}(\omega) = \varepsilon_{\infty} + (\varepsilon_{\mathrm{st}} - \varepsilon_{\infty}) \frac{\omega_0{}^2}{(\omega_0{}^2 - \omega^2 - i\gamma\omega)} \ ,$$

where ω is the angular frequency.

(i) Explain the meaning of the symbols ε_{∞}, $\varepsilon_{\mathrm{st}}$, ω_0 and γ in this formula. **[1]**

(ii) Use the formula to explain why a lightly-damped ionic crystal reflects strongly in a range of infrared frequencies called the Reststrahlen band, stating the upper and lower angular frequency limits of this band. **[3]**

(b) The static and high frequency dielectric constants of InAs are 14.9 and 12.3 respectively. The transverse optic phonons of InAs have a frequency of 6.6×10^{12} Hz.

(i) What is the reflectivity at very low frequencies? **[1]**

(ii) What are the upper and lower wavelengths of the Reststrahlen band? **[1]**

(iii) The reflectivity is found to drop to near zero at a frequency just above the Reststrahlen band. Estimate the wavelength at which this occurs. **[1.5]**

(iv) Estimate the reflectivity at a wavelength of 43 μm, given that $\gamma = 2.5 \times 10^{11}\ \mathrm{s}^{-1}$. **[1.5]**

(v) Account for the large value of γ. **[1]**

END OF EXAMINATION PAPER