DEPARTMENT OF PHYSICS & ASTRONOMY

Autumn Semester 2006-2007

ATOMIC AND LASER PHYSICS **2 Hours**

Answer question ONE (Compulsory) and TWO others.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

THIS QUESTION IS COMPULSORY. Answer all parts

1 (a) Calculate the energy of the $n = 3 \rightarrow 2$ transition in Li^{2+}, expressing your answer in both eV and cm^{-1} units. (The atomic number of lithium is 3.) **[1]**

(b) State, with reasons, which of the following transitions are forbidden for electric dipole radiation:

$$^2P_{1/2} \rightarrow {}^2S_{1/2}$$
$$^4P_{1/2} \rightarrow {}^2S_{1/2}$$
$$^4H_{7/2} \rightarrow {}^4I_{11/2}$$
$$^2F_{5/2} \rightarrow {}^2D_{3/2}$$
$$^3P_2 \rightarrow {}^3D_2$$
$$^3S_1 \rightarrow {}^3S_1$$

[1.5]

(c) Show that the Doppler broadening of a transition of wavelength λ at a temperature T in an atom of mass m is given by

$$\Delta \nu = \frac{2}{\lambda}\left(\frac{2\ln 2\, k_B T}{m}\right)^{1/2},$$

and evaluate the Doppler broadening for the 546 nm line of mercury (relative atomic mass = 200.6) at 250 ºC. **[2]**

(d) The first ionisation potential of the alkali metal cesium (atomic configuration [Xe] 6s) is 3.89 eV. Calculate the quantum defect of the 6s electron in cesium. **[1]**

(e) Draw a diagram showing the energies of the Zeeman-split levels of the $^3P_1$ term of helium relative to the un-split levels at zero field, and state the value of the splitting energies for a magnetic field strength of 2 T. The Landé g factor for an atomic term with quantum numbers L, S and J is given by:

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

[1.5]

(f) Write down the atomic terms that are possible for the (2s,2s) and (2s,3s) electronic configurations of beryllium (Z = 4). **[1]**

(g) A laser contains a Nd:YAG rod of length 5 cm. Find the lowest value of the reflectivity of the output coupler that will still sustain lasing if the maximum value of the gain coefficient in the laser rod is 0.5 m^{-1}. **[1]**

(h) The laser crystal in a mode-locked Ti:sapphire laser has a gain bandwidth extending from 750 nm to 850 nm. Calculate the duration of the shortest pulses that can be produced by this laser. **[1]**

2 The Schrödinger equation for an electron in an atom with a *central potential* is of the form

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right)\psi = E\psi\ .$$

(a) Explain what is meant by a *central potential*, and discuss the importance of the central-field approximation in the treatment of atoms with more than one electron. **[2]**

(b) Show that solutions of the form $\psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$ exist for the Schrödinger equation given above, provided that

$$\left(-\frac{\hbar^2}{2mr^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}}{\mathrm{d}r}\right) + \frac{\hbar^2 l(l+1)}{2mr^2} + V(r)\ \right)R(r) = E\ R(r)\ ,$$

and

$$\left(-\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)Y(\theta,\phi) = l(l+1)\ Y(\theta,\phi)\ ,$$

where (r,θ,ϕ) are the spherical polar co-ordinates and $l(l+1)$ is a separation constant. **[3]**

(c) Show that the solutions of the angular equation must satisfy

$$\hat{L}_z\ Y(\theta,\phi) = m\hbar\, Y(\theta,\phi)$$

and

$$\hat{L}^2\ Y(\theta,\phi) = \hbar^2 l(l+1)\ Y(\theta,\phi)$$

where m is an integer. You may assume without proof that the angular momentum operators are given by

$$\hat{L}_z = -\mathrm{i}\hbar\frac{\partial}{\partial\phi}\ ,$$

and

$$\hat{L}^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right).$$

[3]

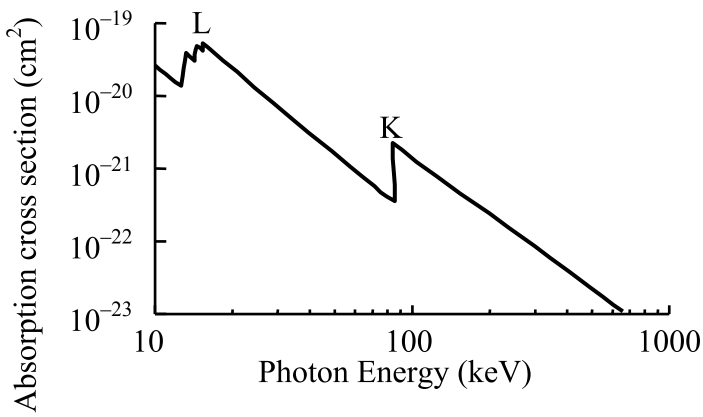
(d) The normalized radial wave function $R(r)$ of the 2p state of hydrogen is given by

$$R(r) = (2/\sqrt{3})\left(1/2a_0\right)^{5/2}\ r\ \exp\left(-r/2a_0\right).$$

Calculate the expectation value of r^{-1}.

You may assume without proof that $\int_0^\infty u^n \mathrm{e}^{-u}\mathrm{d}u = n!$. **[2]**

3



The diagram above shows the X-ray absorption spectrum of lead ($Z = 82$).

(a) Account for the general shape of the spectrum, and discuss how it gives evidence for the shell structure of atoms. **[2]**

(b) The K-shell edge occurs at 88.0 keV. Use this information to deduce the effective charge for the K shell electrons, and explain why this number is not equal to 82. **[2]**

(c) Explain why there is only one absorption edge for the K-shell electrons, but several for the L-shell electrons. **[1]**

(d) Sketch the emission spectrum that you would observe from an X-ray tube containing a lead target when the operating voltage is 30 kV. Your sketch should have wavelength on the horizontal axis, and should state the approximate wavelengths of the M $\rightarrow$ L, N $\rightarrow$ L and M $\rightarrow$ N groups of lines, and also of the short wavelength cut-off. **[4]**

(e) What voltage would have to be applied to this X-ray tube to observe K shell transitions? **[1]**

4 (a) Explain briefly how the observation of the anomalous Zeeman effect gives evidence for the existence of electron spin. **[1]**

(b) Explain why the spin-orbit energy shift of an atom with *LS*-coupling is of the form

$$\Delta E_{\text{spin-orbit}} = C_{\text{FS}}\, \boldsymbol{L}\cdot\boldsymbol{S},$$

where C_{FS} is the same for each of the J terms arising for a particular combination of L and S. In your answer you should state clearly the meanings of $\boldsymbol{L}$ and $\boldsymbol{S}$. **[2]**

(c) Account for the fact that the spin-orbit coupling in an alkali metal causes a splitting of:

(i) p $\rightarrow$ s transitions into doublets,

(ii) d $\rightarrow$ p transition into triplets. **[3]**

(d) Explain the origin of *hyperfine* interactions in atoms and show that this leads to an additional energy shift for each atomic term of the form:

$$\Delta E_{\text{hyperfine}} = C_{\text{HFS}}\, \boldsymbol{I}\cdot\boldsymbol{J}.$$

In your answer you should state clearly the meanings of $\boldsymbol{I}$ and $\boldsymbol{J}$. **[2]**

(e) High resolution spectroscopy on the spectral lines of sodium reveals that the 3p $^2P_{3/2}$ term consists of four hyperfine levels. Show that this is consistent with $I = 3/2$, and deduce the number of hyperfine levels that would be present for the 3s $^2S_{1/2}$ term. **[2]**

5 (a) Write down equations that define the Einstein coefficients A_{21}, B_{21} and B_{12}, and show that, for a transition at frequency ν, the Einstein coefficients are related to each other through

$$g_1 B_{12} = g_2 B_{21} \ ,$$

and

$$A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21} \, ,$$

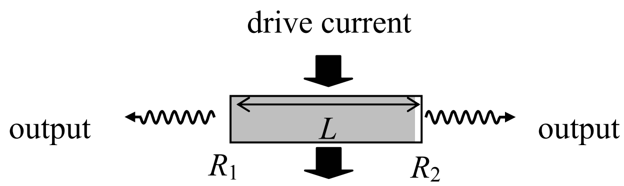
where g_1 and g_2 are the degeneracies of the lower and upper levels. You may assume without proof that the energy density of black-body radiation at temperature T is given by

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp\left(h\nu/kT\right) - 1} \ .$$

[2]

(b) Explain why population inversion is required for laser operation, and explain how population inversion is achieved in **either** a four-level laser **or** a three-level laser. **[3]**

(c) The diagram below shows a schematic diagram of a semiconductor laser diode. A certain diode laser has a length L of 1 mm and has uncoated edges with $R_1 = R_2 = 30\%$. The laser medium has internal losses that can be characterized by a distributed loss coefficient of 200 m^{-1}.



(i) Find the gain coefficient in the laser. **[1]**

(ii) Calculate the frequency separation of the longitudinal modes, given that the refractive index of the laser chip is 3.5. **[1]**

(d) A highly reflective coating is applied to the rear facet of the laser described in part (c), so that $R_1 = 99\%$. Given that the original laser had a threshold current of 100 mA,

(i) what would be the threshold current of the coated laser? (Assume that the gain is proportional to the current below threshold, and that the internal losses are unaffected by the coating of the rear edge of the laser.) **[1.5]**

(ii) How would the power output compare to that of the uncoated laser when both are driven at 200 mA? **[1.5]**

END OF EXAMINATION PAPER