MTH4100 Calculus I

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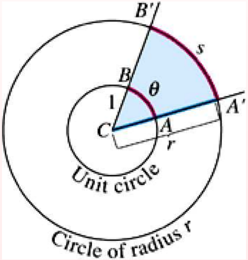
Radians

The **radian measure** of an angle ACB is the length θ of the arc AB on the unit circle.



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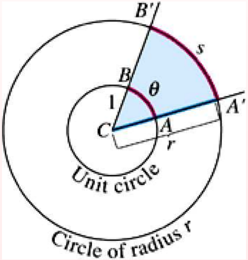

Thus $s = r\theta$ is the length of the arc on a circle of radius r when the angle it subtends θ is measured in radians.

Radians

Conversion formula between degrees and radians:

360° corresponds to 2π, hence:

$$\boxed{\frac{\text{angle in radians}}{\text{angle in degrees}} = \frac{\pi}{180}}$$

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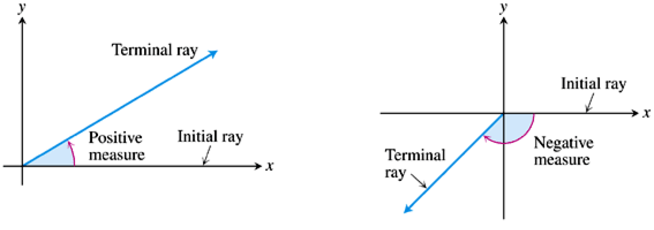
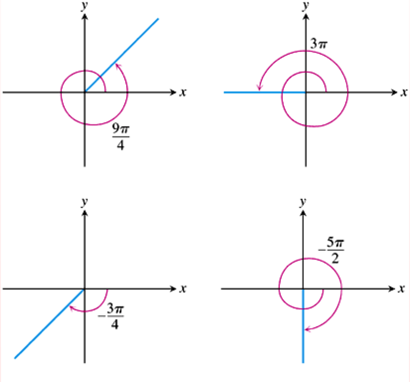
Angles are oriented:

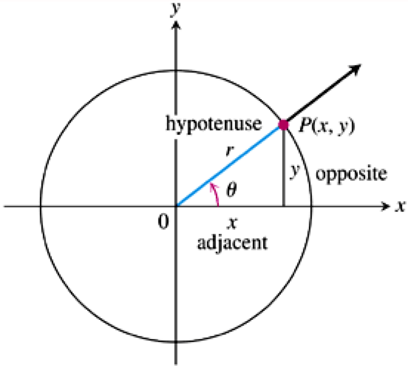
- *positive angle* are measured counter-clockwise;
- *negative angle* are measured clockwise.



Angles can be *larger* (counter-clockwise) or *smaller* (clockwise) than 2π:



Trigonometric functions



sine:	$\sin\theta = \frac{y}{r}$	**cosecant:**	$\csc\theta = \frac{r}{y}$
cosine:	$\cos\theta = \frac{x}{r}$	**secant:**	$\sec\theta = \frac{r}{x}$
tangent:	$\tan\theta = \frac{y}{x}$	**cotangent:**	$\cot\theta = \frac{x}{y}$

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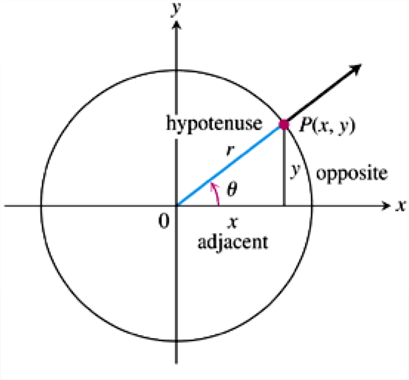

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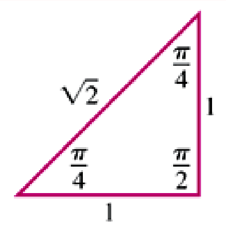
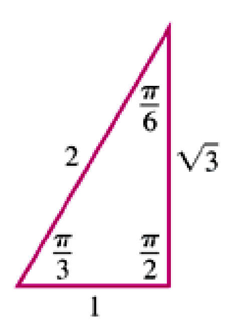
Note that these definitions hold not just for $0 \leq \theta \leq \pi/2$ but for all $\infty < \theta < \infty$.

It is useful to memorize the following two special triangles because *exact values* of the trigonometric functions can be read from them.





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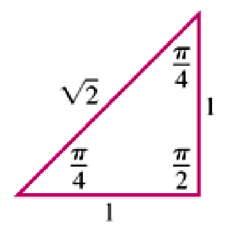
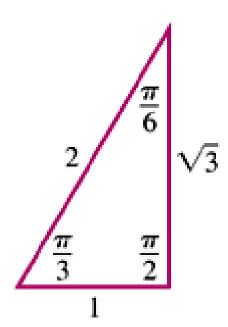




Examples:

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad ; \quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

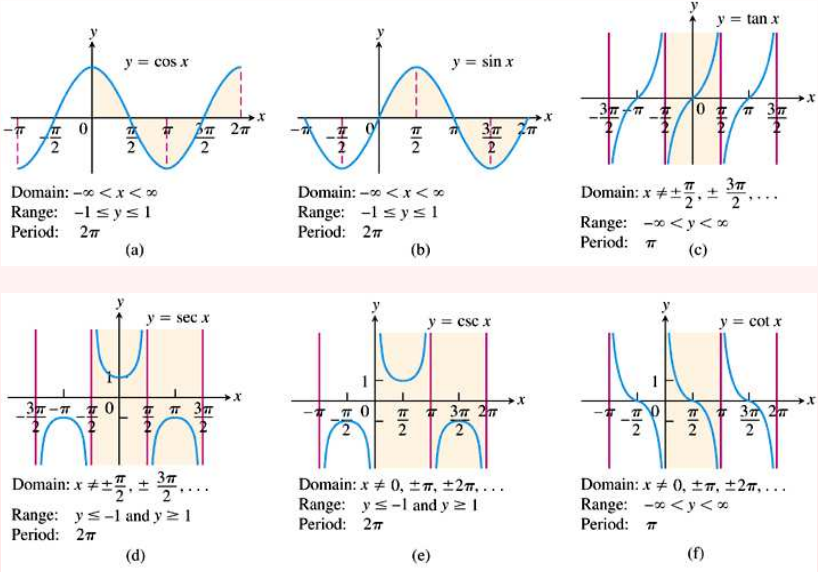
Periodic functions

DEFINITION Periodic Function

A function $f(x)$ is **periodic** if there is a positive number p such that $f(x + p) = f(x)$ for every value of x. The smallest such value of p is the **period** of f.

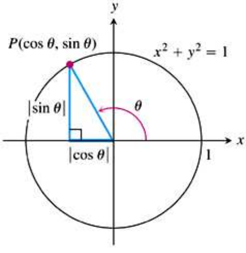
Since for any angle $\theta \in \mathbb{R}$, all six trigonometric functions will take the same value at θ and $\theta + 2\pi$ (why?) all six trigonometric functions are periodic. We can determine their periods by considering their graphs:

Graphs of trigonometric functions



$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

This follows by considering the corresponding triangle inside a unit circle:



An important trigonometric identity

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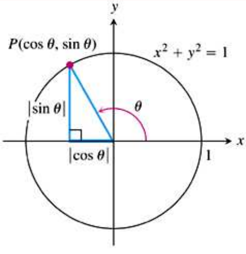

An **identity** is an equation which is valid for *all* values of the variable(s) it contains. The equation $\cos\theta = 1$ is *not* an identity, because it is only true for *some* values of θ, not all.

Another important identity

$$\boxed{\cos(A - B) = \cos A\, cosB + \sin A\ \sin B}$$

We can obtain many other identities for these two identities, see Thomas' Calculus, Section 1.3, p. 25-27.

Read Thomas' Calculus

- short **paragraph** about ellipses, p. 18/19
- **Section 1.3**, p. 25-27 about trigonometric function symmetries and identities

You will need this for Coursework 2.