MTH4100 Calculus I

Bill Jackson
School of Mathematical Sciences QMUL

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- We write *f maps x to $f(x)$* symbolically as $f : x \mapsto f(x)$.

Variables

We often think of the input and output values of a function as *variables*. The function tells us how to determine the value of the output variable y from the value of the input variable x. We write $y = f(x)$ and refer to x as the *independent variable* and y as the *dependent variable*. The function f acts like a "black box" which inputs x and outputs $y = f(x)$.

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Examples:

y is the height of the floor of the lecture hall depending on the distance x from the whiteboard;

y is the stock market index depending on the time x;

y is the volume of a sphere depending on its radius x.

Real functions

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Examples:

Function	**Domain**	**Codomain**	**Range**
$y = x^2$	$(-\infty, \infty)$	$\mathbb{R}$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$\mathbb{R}$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$\mathbb{R}$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$\mathbb{R}$	$[0, 1]$

Remark

A function is fully specified by not only giving the rule f, but also giving its domain D, and its codomain Y. Thus

$$f : \mathbb{R} \to \mathbb{R} \text{ defined by } f : x \mapsto x^2$$

and

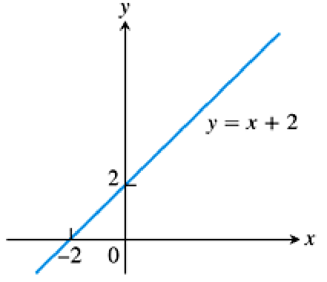
$$g : [0, \infty) \to \mathbb{R} \text{ defined by } g : x \mapsto x^2$$

are *different* functions since they have different domains.

The graph of a function

Definition The *graph* of a function $f : D \to \mathbb{R}$ is of the set of all points $(x, f(x))$ in the cartesian plane whose coordinates are the input-output pairs for f.

Example: $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x + 2$.



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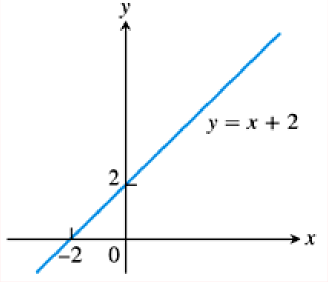

Given a function f, we can *sketch* its graph by plotting some of its points $(x, f(x))$ in the plane and then 'joining them up'. Calculus will help us do this more accurately.

Curves

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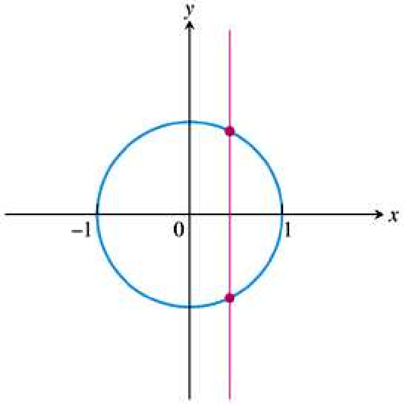
The graph of a function f is a special kind of curve since it is defined by the equation $y = f(x)$. However some curves are not graphs of any function:

Recall that a function f can have only *one value* $f(x)$ *assigned to each* x in its domain. This leads to *the vertical line test*:

No vertical line can intersect the graph of a function *more than once*.

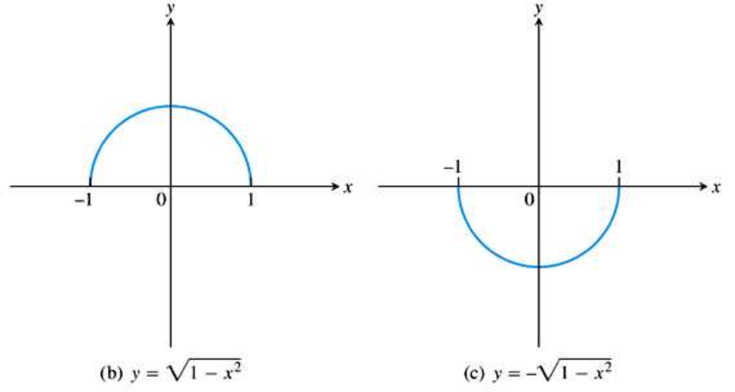
Example



(a) $x^2 + y^2 = 1$

The curve shown in (a) is *not* the graph of a function since it fails the vertical line test.

Example continued



The curves in (b) and (c) *are* graphs of functions.

Piecewise defined functions

Definition A *piecewise defined function* is a function that is described by using different formulas on different parts of its domain.

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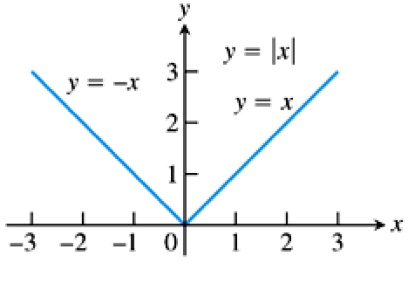
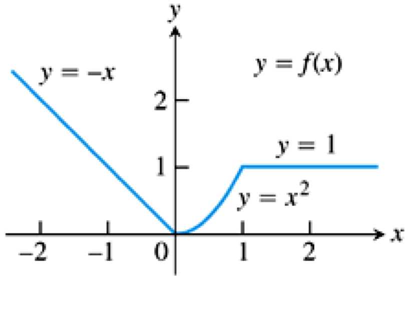
Example: The absolute value function

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Another example

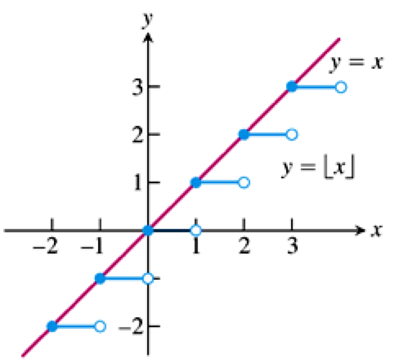
$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$$



The *floor function*

$$f(x) = \lfloor x \rfloor$$

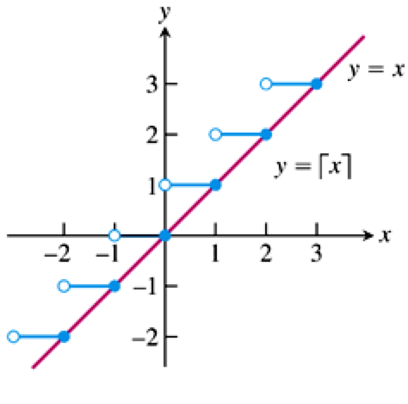
is defined by taking $\lfloor x \rfloor$ to be the greatest integer which is less than or equal to x. Thus $\lfloor 1.3 \rfloor = 1$ and $\lfloor -2.7 \rfloor = -3$.



The *ceiling function*

$$f(x) = \lceil x \rceil$$

is defined by taking $\lceil x \rceil$ to be the smallest integer which is greater than or equal to x. Thus $\lceil 3.5 \rceil = 4$ and $\lceil -1.8 \rceil = -1$.



Some important functions

- **linear function:** $\boxed{f(x) = mx + b}$ for some $m, b \in \mathbb{R}$

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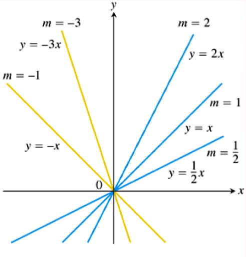
When $b = 0$, $f(x) = mx$ and the graph of f is a line through the origin.



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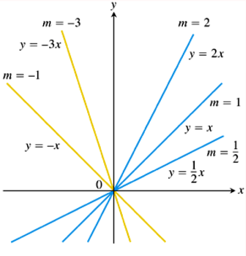
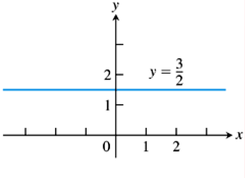


When $m = 0$, $f(x) = b$ and f is a *constant function*.



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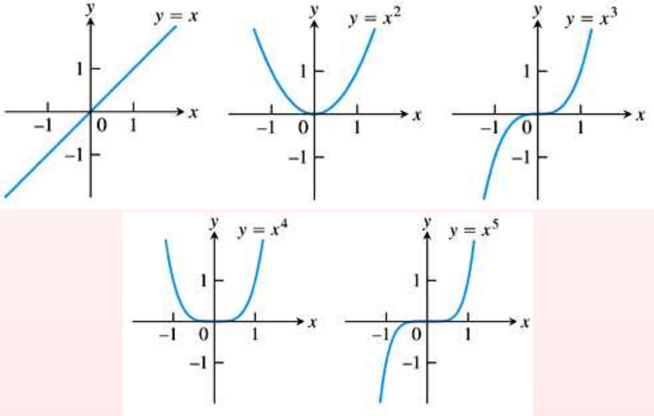
Graphs of $f(x) = x^a$ for $a = 1, 2, 3, 4, 5$



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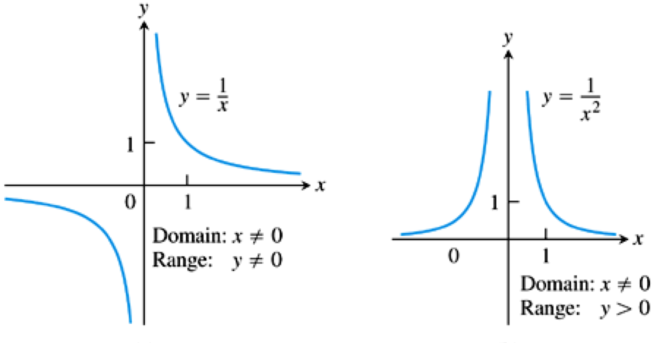
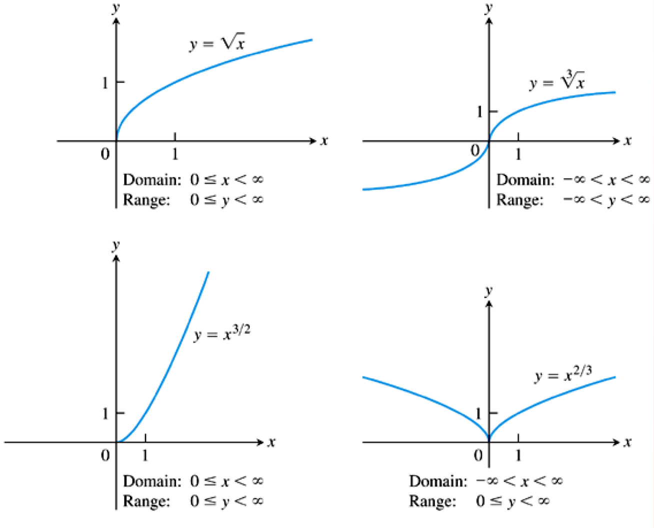


Graphs of $f(x) = x^a$ for $a = \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{2}{3}$



Some important functions

- **polynomial function:** $p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ for $n \in \mathbb{Z}$ with $n \geq 0$, and $a_0, a_1, \ldots, a_{n-1}, a_n \in \mathbb{R}$ with $a_n \neq 0$. We say that: $p(x)$ is a *polynomial in* x; $a_0, a_1, \ldots, a_{n-1}, a_n \in \mathbb{R}$ are the *coefficients* of $p(x)$; n is the *degree* of $p(x)$.

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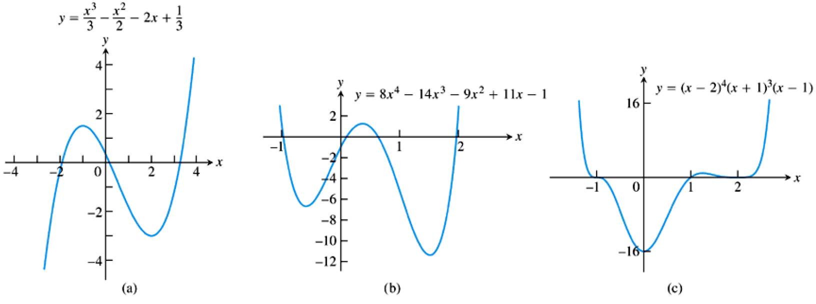
Three polynomial functions and their graphs



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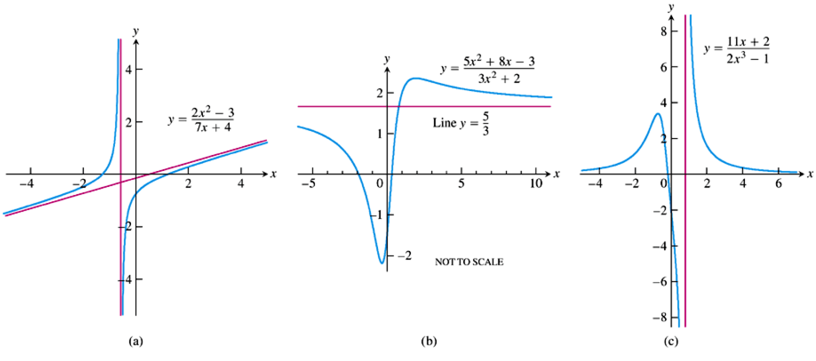
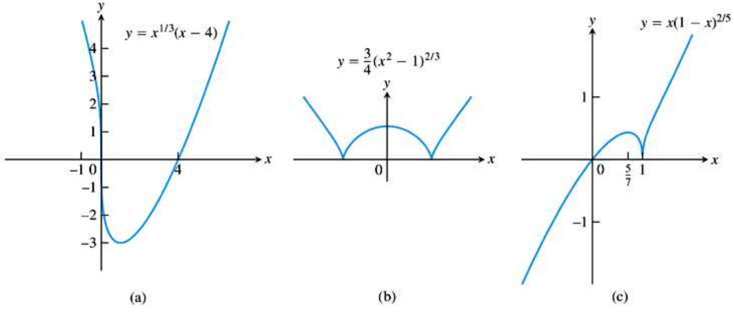
Three rational functions and their graphs



We will see many other important functions throughout this module. For example:

algebraic functions: any function constructed from polynomials using algebraic operations (including taking roots):



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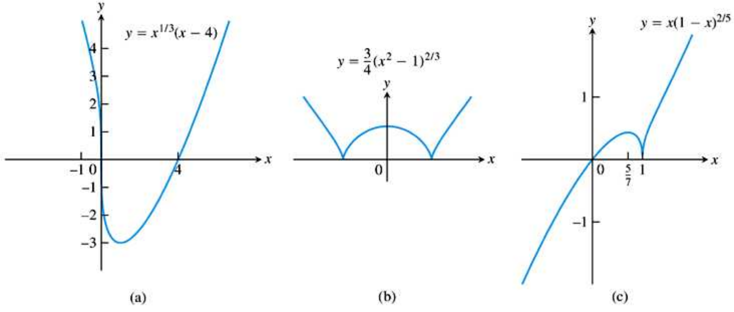

trigonometric functions

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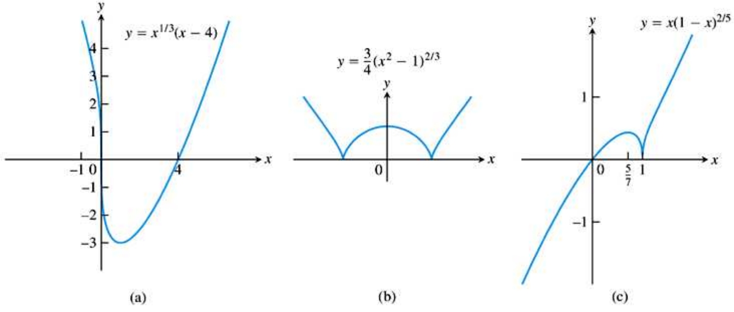

trigonometric functions

exponential and **logarithmic functions**

Special kinds of functions

Definition A function $f : D \to \mathbb{R}$ is *increasing* on some interval $I \subseteq D$ if $f(x_1) \leq f(x_2)$ whenever $x_1, x_2 \in I$ and $x_1 \leq x_2$. (Informally, f is increasing if the graph of f "climbs" or "rises" as we move along I from left to right.)

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Examples:

function	**where increasing**	**where decreasing**
$y = x^2$	$0 \leq x < \infty$	$-\infty < x \leq 0$
$y = 1/x$	nowhere	$-\infty < x < 0$ and $0 < x < \infty$
$y = 1/x^2$	$-\infty < x < 0$	$0 < x < \infty$
$y = x^{2/3}$	$0 \leq x < \infty$	$-\infty < x \leq 0$

Special kinds of functions

Definition A function $f : \mathbb{R} \to \mathbb{R}$ is *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$. (This is the same as saying its graph is symmetric about the y-axis.)

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Similarly, f is *odd* if $f(-x) = -f(x)$ for $x \in \mathbb{R}$. (This is the same as saying its graph is symmetric about the origin.) **Examples:**
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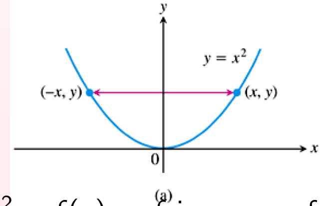
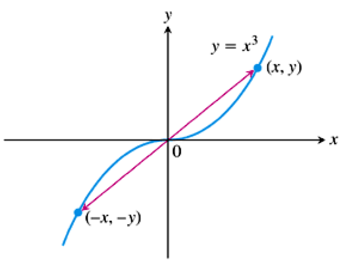


$f(-x) = (-x)^2 = x^2 = f(x)$ so f is an even function; its graph is *symmetric about the y-axis*.

Even and odd functions - examples

(b) $f(x) = x^3$

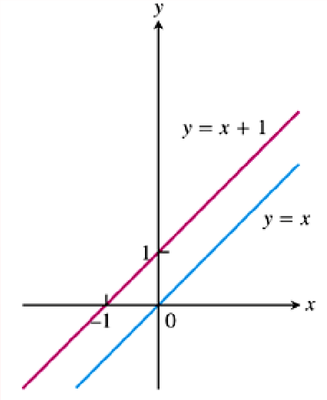


(b)

$f(-x) = (-x)^3 = -x^3 = -f(x)$: odd function; its graph is symmetric about the origin.

Even and odd functions - examples

(c) $f(x) = x$ and $g(x) = x + 1$



$f(-x) = -x = -f(x)$ so f is an odd function

$g(-x) = -x + 1 \neq g(x)$ and $-g(x) = -x - 1 \neq g(-x)$ so g is neither even nor odd.

Algebraic combinations of functions

Suppose $f : D \to \mathbb{R}$ and $g : E \to \mathbb{R}$ are functions. Then we can define new functions $f + g$, $f - g$ and fg with domain $D \cap E$ as follows:

$$(f + g)(x) = f(x) + g(x)$$
$$(f - g)(x) = f(x) - g(x)$$
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We refer to these new functions as the *sum, difference, product, and quotient* of f and g.

Algebraic combinations of functions - examples

Examples:

$$f(x) = \sqrt{x} \quad \text{domain } D = [0, \infty)$$

$$g(x) = \sqrt{1 - x} \qquad \text{domain } E = (-\infty, 1]$$

intersection of both domains:

$$D \cap E = [0, \infty) \cap (-\infty, 1] = [0, 1]$$

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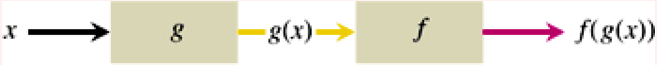
function	formula	domain
$f+g$	$(f+g)(x) = \sqrt{x} + \sqrt{1-x}$	$[0, 1]$
$f-g$	$(f-g)(x) = \sqrt{x} - \sqrt{1-x}$	$[0, 1]$
$g-f$	$(g-f)(x) = \sqrt{1-x} - \sqrt{x}$	$[0, 1]$
fg	$(fg)(x) = f(x)g(x) = \sqrt{x(1-x)}$	$[0, 1]$
f/g	$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \sqrt{\frac{x}{1-x}}$	$[0, 1)$ ($x = 1$ excluded)
g/f	$\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \sqrt{\frac{1-x}{x}}$	$(0, 1]$ ($x = 0$ excluded)

Composition of functions

Definition Suppose $f : D \to \mathbb{R}$ and $g : E \to R$ are functions. Then the *composite* function $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x)).$$

(We read $f \circ g$ as "f composed with g". We also refer to $f \circ g$ as "the composition of f with g.")



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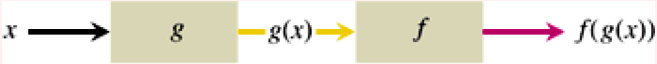

The *domain* of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f, i.e.
$\{x \in \mathbb{R} : x \in E \text{ and } g(x) \in D\}$.

Composition of functions - example

(a) Suppose

$$\begin{array}{llllll} f(x) & = \sqrt{x} & \text{domain} & D = [0, \infty) & \text{range} & R = [0, \infty) \\ g(x) & = x + 1 & \text{domain} & E = (-\infty, \infty) & \text{range} & S = (-\infty, \infty) \end{array}$$

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Then

composite	**domain**
$(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x+1}$	$[-1, \infty)$
$(g \circ f)(x) = g(f(x)) = f(x) + 1 = \sqrt{x} + 1$	$[0, \infty)$
$(f \circ f)(x) = f(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{1/4}$	$[0, \infty)$
$(g \circ g)(x) = g(g(x)) = g(x) + 1 = x + 2$	$(-\infty, \infty)$

Composition of functions - example

(a) Suppose

$f(x) = \sqrt{x}$	domain	$D = [0, \infty)$	range	$R = [0, \infty)$
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Then

composite	**domain**
$(f \circ g)(x) = \lvert x \rvert$	$(-\infty, \infty)$
$(g \circ f)(x) = x$	$[0, \infty)$

Shifting the graph of a function

Suppose f is a function and $c \in \mathbb{R}$. Let g and h be two new functions defined by $g(x) = f(x) + c$ and $h(x) = f(x + c)$. Then

- the graph of g is equal to the graph of f shifted up by c units.
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Note that if $c < 0$ then a shift up by c units is actually a shift down, and a shift to the left by c units is actually a shift to the right.

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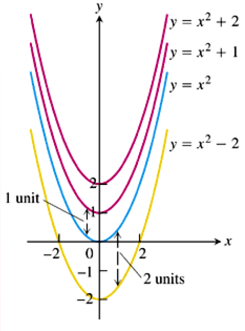
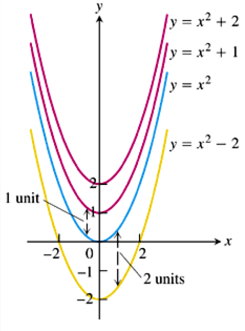
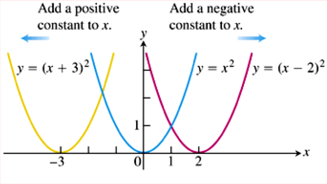
Note also that g and h can both be obtained from f by taking a composition with a linear function: if $k(x) = x + c$ for all $x \in \mathbb{R}$ then $g = k \circ f$ and $h = f \circ k$.

Shifting the graph of a function - Example



Shifting the graph of a function - Example





Scaling the graph of a function

Suppose f is a function and $c\mathbb{R}$. Let g and h be two new functions defined by $g(x) = cf(x)$ and $h(x) = f(cx)$. If $c > 0$ then

- the graph of g is equal to the graph of f scaled by a factor of c along the y-axis.
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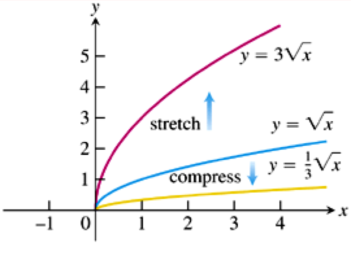
Example $y = \sqrt{x}$



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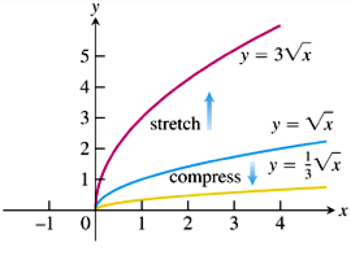

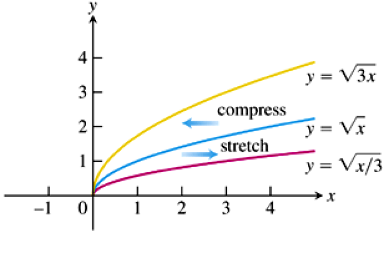

Reflecting the graph of a function

If $c = -1$ i.e. $g(x) = -f(x)$ and $h(x) = f(-x)$, then

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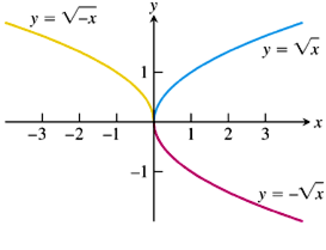

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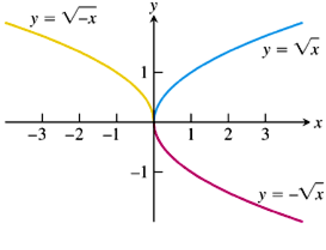

If $c < 0$ is an arbitrary negative real number then we obtain a combination of a scaling and a reflection: see Exercise Sheet 2 for examples