Queen Mary
University of London

MTH4101   **Calculus II, Spring 2013**
Problem sheet for Tutorial 9   **Rainer Klages**

- The questions are designed to help you with material covered in Week 10. You will get help with them in the tutorial on 21 or 22 March.

- For this final exercise sheet no written solutions have to be handed in. *Nevertheless, it is important that you do all the exercises.* Please note that all material covered in this module up to, and including, week 12, as well as all corresponding exercises, are important, and relevant for the exam. Model solutions to this exercise sheet will be available after week 12 on the course webpage.

1: Evaluate the following integral by changing to polar coordinates:

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) \, dx \, dy \ .$$

(*) 2: Solve the system $u = 2x + y$, $v = x - y$ to find expressions for x and y in terms of u and v. Use these expressions to find the Jacobian $\partial(x,y)/\partial(u,v)$. Hence evaluate the integral

$$\int \int_R (2x^2 - xy - y^2) \, dx \, dy$$

for the region R bounded by the lines $y = -2x + 1$, $y = -2x + 3$, $y = x$, $y = x - 1$. [2009 exam question]

3: Evaluate the following triple integrals:

(a) $\displaystyle \int_1^e \int_1^e \int_1^e \frac{1}{xyz} \, dx \, dy \, dz;$ (b) $\displaystyle \int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz \, dy \, dx.$